Discussiones Mathematicae
Probability and Statistics 37 (2017) 101–103
doi:10.7151/dmps.1190

GAUSSIAN MIXTURES AND FINANCIAL RETURNS

C. Cuevas-Covarrubias, J. Iñigo-Martínez

and

R. Jímenez-Padilla

Centro de Investigación en Estadística y Matemáticas Aplicadas
Facultad de Ciencias Actuariales
Universidad Anáhuac México

e-mail: ccuevas@anahuac.mx

Abstract

Many important models in quantitative finance are based on the assumption that stock returns are independent and normally distributed. However, the empirical distributions of price changes are frequently skewed and leptokurtic. Therefore, flexible distributions and their potential in financial modeling constitute an important research topic in mathematical finance. We explore the potential of Gaussian mixtures as an alternative to the normal distribution. Our discussion is based on three practical examples taken from the Mexican stock market. This article is not limited to the estimation of marginal distributions. Contrasting with some other papers in the literature, the application of multivariate Gaussian mixtures to estimate joint distributions of financial returns is also analyzed. This multivariate approach gives us the opportunity to illustrate the application of Gaussian mixtures in portfolio theory and risk assessment.

Keywords: Gaussian mixtures, financial returns, VaR, CVaR.

2010 Mathematics Subject Classification: 62-09, 62P05, 62F10, 91G70.

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Received 23 June 2017
Accepted 17 August 2017