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THE "THIRTY-SEVEN PERCENT RULE" AND THE SECRETARY PROBLEM WITH RELATIVE RANKS

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Abstract

We revisit the problem of selecting an item from n choices that appear before us in random sequential order so as to minimize the expected rank of the item selected. In particular, we examine the stopping rule where we reject the first k items and then select the first subsequent item that ranks lower than the l-th lowest-ranked item among the first k. We prove that the optimal rule has $k \sim n/\mathrm{e}$, as in the classical secretary problem where our sole objective is to select the item of lowest rank; however, with the optimally chosen l, here we can get the expected rank of the item selected to be less than any positive power of n (as n approaches infinity). We also introduce a common generalization where our goal is to minimize the expected rank of the item selected, but this rank must be within the lowest d.

Keywords: secretary problem, relative ranks, stopping rule, optimization.

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