Discussiones Mathematicae
Probability and Statistics 32 (2012) 47–49
doi:10.7151/dmps.1143

NORMALITY ASSUMPTION FOR THE LOG-RETURN OF THE STOCK PRICES

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Abstract

The normality of the log-returns for the price of the stocks is one of the most important assumptions in mathematical finance. Usually is assumed that the price dynamics of the stocks are driven by geometric Brownian motion and, in that case, the log-return of the prices are independent and normally distributed. For instance, for the Black-Scholes model and for the Black-Scholes pricing formula [4] this is one of the main assumptions. In this paper we will investigate if this assumption is verified in the real world, that is, for a large number of company stock prices we will test the normality assumption for the log-return of their prices. We will apply the Kolmogorov-Smirnov [10, 5], the Shapiro-Wilks [17, 16] and the Anderson-Darling [1, 2] tests for normality to a wide number of company prices from companies quoted in the Nasdaq composite index.

Keywords: Anderson-Darling, Black-Scholes, Geometric Brownian motion, Kolmogorov-Smirnov, Log-return, Normality test, Shapiro-Wilks.

2010 Mathematics Subject Classification: 62F03, 62F10, 62G30, 62Q05.

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[1]This work was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through PEst-OE/MAT/UI0297/2011 (CMA)

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Received 18 May 2012