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NEAR-EXACT DISTRIBUTIONS FOR THE GENERALIZED WILKS LAMBDA STATISTIC

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Dedicated to Professor J. Tiago Mexia
on his Jubilee

Abstract

Two near-exact distributions for the generalized Wilks Lambda statistic, used to test the independence of several sets of variables with a multivariate normal distribution, are developed for the case where two or more of these sets have an odd number of variables. Using the concept of near-exact distribution and based on a factorization of the exact characteristic function we obtain two approximations, which are very close to the exact distribution but far more manageable. These near-exact distributions equate, by construction, some of the first exact moments and correspond to cumulative distribution functions which are practical to use, allowing for an easy computation of quantiles. We also develop three asymptotic distributions which also equate some of the first exact moments. We assess the proximity of the asymptotic and near-exact distributions obtained to the exact distribution using two measures based on the Berry-Esseen bounds. In our comparative numerical study we consider different numbers of sets of variables, different numbers of variables per set and different sample sizes.

Keywords: independent Beta random variables, characteristic function, sum of Gamma random variables, likelihood ratio test statistic, proximity measures.

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