*Discussiones Mathematicae*
*Probability and Statistics 30 (2010) 21–33*
*doi:10.7151/dmps.1119*

# EXTREMAL BEHAVIOUR OF STATIONARY PROCESSES: THE CALIBRATION TECHNIQUE IN THE EXTREMAL INDEX ESTIMATION

D. Prata Gomes

*CMA and Mathematics Department, Faculty of Science and Technology*
*New University of Lisbon*
*Monte de Caparica 2829–516 Caparica, Portugal*

**e-mail:** dsrp@fct.unl.pt

AND

Maria Manuela Neves

*CEAUL and Mathematics Department, Instituto Superior de Agronomia,*
*Technical University of Lisbon*
*Tapada da Ajuda, 1349–017, Lisboa, Portugal*

**e-mail:** manela@isa.utl.pt

**Abstract**

Classical extreme value methods were derived when the underlying process is assumed to be a sequence of independent random variables. However when observations are taken along the time and/or the space the independence is an unrealistic assumption. A parameter that arises in this situation, characterizing the degree of local dependence in the extremes of a stationary series, is the extremal index, $\theta$. In several areas such as hydrology, telecommunications, finance and environment, for example, the dependence between successive observations is observed so large values tend to occur in clusters. The extremal index is a quantity which, in an intuitive way, allows one to characterise the relationship between the dependence structure of the data and their extremal behaviour. Several estimators have been studied in the literature, but they endure a problem that usually appears in semiparametric estimators - a strong dependence on the high level $u_n$, with an increasing bias and a decreasing variance as the threshold decreases.

The calibration technique (Scheffé, 1973) is here considered as a procedure of controlling the bias of an estimator. It also leads to the construction of confidence intervals for the extremal index. A simulation study was performed for a stationary sequence and two sets of stationary data are under study for applying this technique.

**Keywords:** extreme value, stationary sequences, extremal index, estimation, calibration technique.

**2000 Mathematics Subject Classification:** 62G32, 62G09, 62G05, 62J05.

## References

[1] F. Andrews, *Calibration and statistical inference*, J. Ann. Statist. Assoc. **65** (1970), 1233–1242. doi:10.1080/01621459.1970.10481158

[2] S.G. Coles, J.A. Tawn and R.L. Smith, *A sazonal Markov model for extremely low temperatures*, Environmetrics **5** (1994), 221–339. doi:10.1002/env.3170050304

[3] P. Deheuvels, *Point processes and multivariate extreme values*, J. Multivariate Analysis **13** (1983), 257–272. doi:10.1016/0047-259X(83)90025-8

[4] M.I. Gomes, *Statistical inference in an extremal markovian model*, COMPSTAT (1990), 257–262.

[5] M.I. Gomes, *Modelos extremais em esquemas de dependência*, I Congresso Ibero-Americano de Esdadistica e Investigacion Operativa (1992), 209–220.

[6] M.I. Gomes, *On the estimation of parameters of rare events in environmental time series*, *Statistics for the Environment* (1993), 226–241.

[7] T. Hsing, *Extremal index estimation for weakly dependent stationary sequence*, Ann. Statist **21** (1993), 2043–2071. doi:10.1214/aos/1176349409

[8] M.R. Leadbetter, G. Lindgren and H. Rootzen, *Extremes and related properties of random sequences and series*, Springer Verlag. New York 1983. doi:10.1007/978-1-4612-5449-2

[9] S. Nandagopalan, *Multivariate Extremes and Estimation of the Extremal Index*, Ph.D. Thesis. Techn. Report 315, Center for Stochastic Processes, Univ. North-Caroline 1990.

[10] D. Prata Gomes, Métodos computacionais na estimação pontual e intervalar do índice extremal. Tese de Doutoramento, Universidade Nova de Lisboa, Faculdade de Cięncias e Tecnologia 2008.

[11] H. Scheffé, *A statistical theory of calibration*, *Ann. Statist* **1** (1973), 1–37. doi:10.1214/aos/1193342379

[12] R. Smith and I. Weissman, *Estimating the extremal index*, J. R. Statist. Soc. B, **56** (1994), 515–528.

[13] I. Weissman and S. Novak, *On blocks and runs estimators of the extremal index*, J. Statist. Plann. Inf. **66** (1998), 281–288. doi:10.1016/S0378-3758(97)00095-5

[14] E.J. Williams, *Regression methods in calibration problems*, Proc. 37th Session, Bull. Int. Statist. Inst. **43** (1) (1969), 17–28.

Received 27 January 2010