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PREMIUM EVALUATION FOR DIFFERENT LOSS DISTRIBUTIONS USING UTILITY THEORY

Harman Preet Singh Kapoor

and

Kanchan Jain

Department of Statistics
Panjab University, Chandigarh-160014, India

e-mail: harman.pu.87@gmail.com
e-mail: jaink14@gmail.com

Abstract

For any insurance contract to be mutually advantageous to the insurer and the insured, premium setting is an important task for an actuary. The maximum premium (P_{max}) that an insured is willing to pay can be determined using utility theory. The main focus of this paper is to determine P_{max} by considering different forms of the utility function. The loss random variable is assumed to follow different Statistical distributions viz Gamma, Beta, Exponential, Pareto, Weibull, Lognormal and Burr. The theoretical expressions have been derived and the results have also been depicted graphically for some values of distribution parameters.

Keywords: utility function, insurance, premium, loss distribution.

2010 Mathematics Subject Classification: 62P05.

1. Introduction

The most important thing for insurance companies is the setting of premiums for different types of policies or different levels of risk. Premium is affected by many factors like age, sex, moral hazard and utility function. In Actuarial Science, many different premium principles have been proposed (ref. Goovaerts, De Vijlder and Haezendonck, 1984). Wang (1995, 1996) and Wang and Young (1997) and other authors proposed pricing insurance risks using a distortion function. There exits an economic theory that describes the reasons why insured are willing to pay a premium more than the mathematical expectation of their loss, that is, the net premium (Kaas, Goovaerts, Dhaene and Denuit, 2004). These approaches to pricing insurance contracts treat insurance losses as positive random variables and produce premium that are higher than the expected value of the insurance loss. The existence of the insurance industry depends upon investor's willingness to pay for being insured. It must, however, be admitted that the modern use of the utility concept in insurance literature is due to the results given by Neumann and Morgenstern (1944). The expected utility theory became popular after these authors developed their axiomatic approach in 1947. Borch (1974, 1990) explained the relevance of the expected utility theory in order to solve problems in insurance. As Trowbridge (1989) pointed out, utility theory can be seen as the philosophical basis of actuarial science. For more details regarding expected utility, the interested readers can refer to Huang and Litzenberger (1988), Schmidt (1998), Panjer (1998), Kaas, Goovaerts, Dhaene and Denuit (2005) and the references therein.

The utility function $u(w)$ is defined on a set of prospects and represents preferences over these prospects. The utility function satisfies the principle of non-satiation,that is, $u'(w) > 0$. This means that $u(w)$ is an increasing function of wealth w and people prefer more wealth to less. In insurance and finance sector, the investor preferences are assumed to be influenced by their attitude towards risk, which can be expressed in terms of properties of utility functions. Investors can be risk-averse, risk-neutral or risk-seeking (ref. Dickson, 2005). A risk-averse (risk-seeking) investor values an incremental increase (decrease) in wealth less highly than an incremental decrease (increase). For a risk averse (risk seeking) investor, the utility function $u(w)$ is strictly concave (convex), that is, $u''(w) < (>)0$, A risk- neutral investor is indifferent towards risk and for him,$u'(w) > 0$ and $u''(w) = 0$. The form of the utility function can be chosen to model an individual's preferences according to whether or not, he likes, dislikes or is indifferent to risk. The higher the curvature of $u(w)$, the higher will be the risk aversion. However, since expected utility functions are not uniquely defined, a measure that stays constant is the Arrow-Pratt measure of absolute risk-aversion (ARA) (ref. Arrow, 1971) and (Pratt, 1964). This is defined as $A(w) = -\frac{u''(w)}{u'(w)}$. Decreasing/increasing absolute risk aversion (DARA/IARA) is present if $A(w)$ is decreasing/increasing. IARA (DARA) implies that the utility function is positively skewed, that is, $u''(w) < (>) 0$ (Ref. Haim, 2006).

Different types of utility functions prevailing in literature (Kaas *et al.*, 2004) and (Dickson, 2005) are

Linear utility function: $u(w) = w$;

Exponential utility function: $u(w) = -ae^{-aw}$, $a > 0$;

Quadratic utility function: $u(w) = -(a - w)^2$, $w \leq a$;

Fractional Power utility function: $u(w) = w^c$, $0 < c < 1$.

Among the above-mentioned utility functions, the linear utility function corresponds to a risk-neutral investor. The risk-averse investor prefers to use exponential or quadratic or fractional power utility function. It is normally assumed that most of the investors are risk averse. Consequently, they accept additional risk (or are ready to pay higher premium than the expected amount of loss) only if they expect a higher level of return. Only a few of the investors are risk-neutral and risk-seeking investors are not very common. Moreover, exponential utility function is unique in exhibiting constant absolute risk aversion (CARA). An example of a DARA utility function is the logarithmic function and quadratic utility function is an IARA utility function. In the light of the above discussion, we focus on study of linear, exponential, quadratic and fractional power utility function.

If $u(w)$ is the utility function of a decision maker with wealth w, then he makes a choice between random losses X and Y by comparing the expected utilities $E(u(w - X))$ and $E(u(w - Y))$ and choosing the loss with higher expected utility. The utility theory helps the insured (with initial wealth w) determine the maximum premium P_{max} that he is prepared to pay for a random loss X. According to the expected value principle, P_{max} satisfies the utility equilibrium equation (Kaas *et al.*, 2001) and (Dickson, 2005) given as

$$E[u(w - X)] = u(w - P_{max}). \tag{1}$$

The minimum premium (P_{min}) acceptable by the insurer, can be evaluated using the utility equilibrium equation given by $E[u(w + P_{min} - X)] = u(w)$ (Kaas *et al.*, 2005) where $u(w)$ is the utility function of the insurer with initial wealth w.

If P_{max} is greater than P_{min}, the utilities of both the parties increase if the premium paid lies between P_{min} and P_{max} (Kaas *et al.*, 2004). Further an insurance contract with expected loss $E(X)$ is feasible if $P_{max} \geq P_{min} \geq E(X)$ (Bowers, Gerber, Hickman, Jones and Nesbitt, 1997).

In this paper, our basic interest is to evaluate the maximum premium that the insured is willing to pay to the insurer, using the Expected utility model. The behaviour of the premium amount is explored for different combinations of loss distributions and utility functions. Theoretical expressions are derived for the maximum premium. Conclusions are drawn about the impact of P_{max} on insured or insurer based on the values of the parameters of the loss distribution.

In Section 2, the expressions for P_{max} are derived assuming the linear form of utility function when the loss random variable X follows Gamma or Beta or Exponential or Pareto or Weibull or Lognormal or Burr Distributions. For the above mentioned distributions, Sections 3, 4 and 5 deal with derivation of expressions for P_{max} when the utility functions are quadratic, exponential and fractional power. Section 6 compares the values of P_{max} for different utility functions and loss distributions Gamma, Pareto and Weibull with similar values of the parameters.

2. Determination of maximum premium for linear utility function

We consider combinations of linear utility function $u(w) = w$ and different loss distributions to evaluate the maximum premium. The probability density function $f(x)$ of the loss random variable X is assumed to exist. Solving (1) yields

$$P_{max} = E(X). \tag{2}$$

Using (2), the expressions for P_{max} are displayed in the following table.

Table 1. Maximum premium for linear utility function.

Distribution	P_{max}
Gamma (α, β)	$\alpha\beta, \quad \alpha > 0,\ \beta > 0$
Beta (α, β)	$\alpha/(\alpha+\beta), \quad \alpha > 0,\ \beta > 0$
Exponential (λ)	$1/\lambda, \quad \lambda > 0$
Pareto (α, θ)	$\frac{\alpha\theta}{\theta-1}, \quad \alpha > 0,\ \ \theta > 1$
Weibull (α, β)	$\beta\Gamma\big(1+\frac{1}{\alpha}\big), \quad \alpha > 0,\ \beta > 0$
Lognormal (μ, σ)	$\exp\big(\mu+\dfrac{\sigma^2}{2}\big), \ \ 0 < \mu < \infty, \sigma > 0$
Burr $(\alpha, \gamma, \lambda)$	$\dfrac{\Gamma\big(\alpha-\frac{1}{\mu}\big)\Gamma\big(1+\frac{1}{\mu}\big)\lambda^{\frac{1}{\mu}}}{\Gamma\,(\alpha)}, \quad \lambda > 0, \ \ \alpha\mu > 1$

The following figures depict the trend of maximum premium P_{max} for different loss distributions with different combinations of parametric values of the loss distribution.

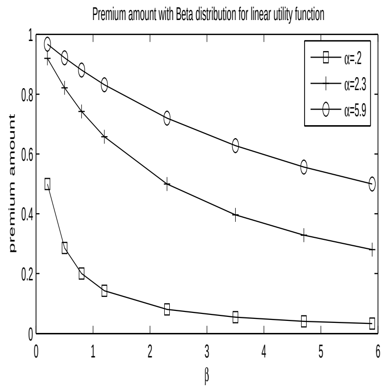

Figure 1. Beta Distribution B(α, β)

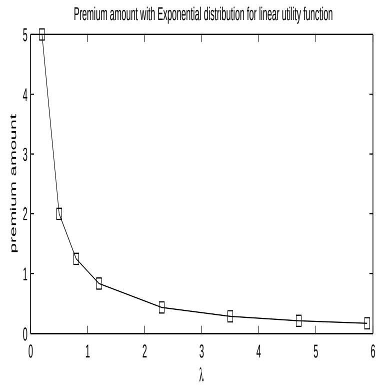

Figure 2. Exponential Distribution

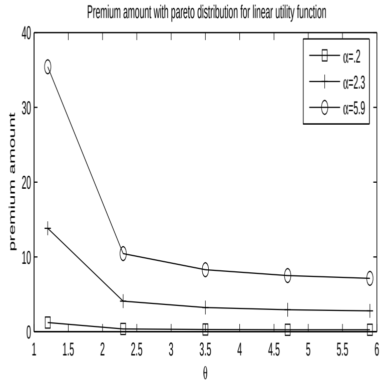

Figure 3. Pareto Distribution

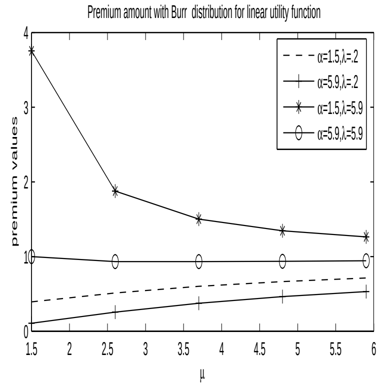

Figure 4. Burr Distribution

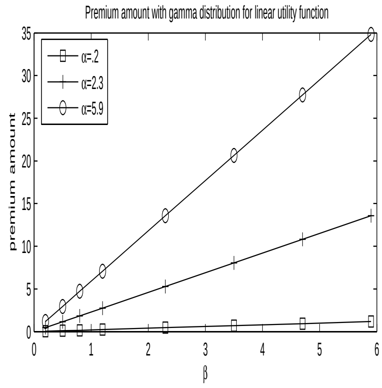

Figure 5. Gamma Distribution

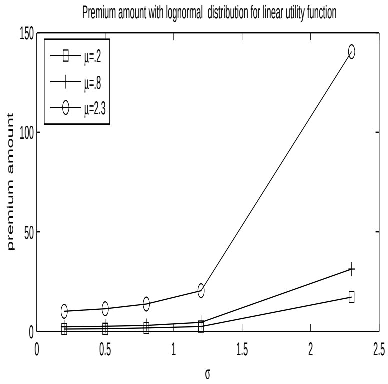

Figure 6. Lognormal Distribution

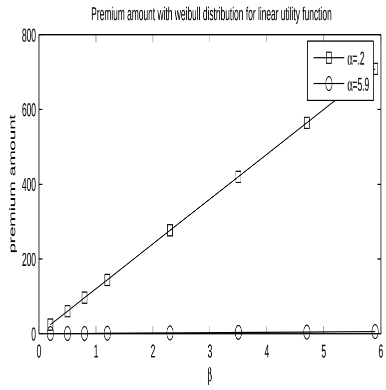

Figure 7. Weibull Distribution

If the insured adopts a linear utility function, then from the above figures, we conclude that

1. for loss distributions Gamma, Lognormal and Weibull, the maximum premium increases as parameters of distributions increase. From insurer's point of view, it is beneficial to use the above mentioned loss distributions with large values of parameters,
2. if insured's loss follows Exponential, Beta or Pareto distribution, the maximum premium declines as parametric values increase. This situation is beneficial for the insured as he will have to be ready to pay less premium,
3. with Burr as loss distribution with α, $\mu > 1$, the maximum premium
   (a) increases for $\lambda \in (0, 1)$,
   (b) decreases for $\lambda > 1$,
   (c) stabilises for higher values of parameters α, λ and μ.

3. Determination of maximum premium for quadratic utility function

For quadratic utility function, $u(w) = -(a - w)^2$, $w \leq a$, the premium P_{max} is found to be a solution of the equation

$$P_{max}{}^2 + 2(a - w)P_{max} - 2(a - w)E(X) - E(X^2) = 0. \tag{3}$$

The following table compiles the theoretical expressions of P_{max} for different loss distributions.

The following figures depict the trend of maximum premium P_{max} for different loss distributions with different combinations of parametric values. Here we assume that initial wealth $w = 100$ and $a = 105$ so that $a > w$.

Table 2. Maximum premium for quadratic utility function.

Distribution	P_{max}
Gamma (α, β)	$-a + w \mp \sqrt{a^2 + 2a\alpha\beta + \alpha(\alpha+1)\beta^2 - 2aw - 2\alpha\beta w + w^2}$, $\alpha > 0, \beta > 0$
Beta (α, β)	$-a + w \mp \sqrt{a^2 + 2a\frac{\alpha}{(\alpha+\beta)} + \frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)} - 2aw - 2\frac{\alpha}{(\alpha+\beta)}w + w^2}$, $\alpha > 0, \beta > 0$
Exponential (λ)	$-a + w \mp \sqrt{a^2 + 2a\frac{1}{\lambda} + \frac{2}{\lambda^2} - 2aw - 2\frac{1}{\lambda}w + w^2}, \lambda > 0$
Pareto (α, θ)	$-a + w \mp \sqrt{a^2 + 2a\,\frac{\alpha\theta}{\theta-1} + \frac{\alpha^2\theta}{(\theta-2)} - 2aw - 2\frac{\alpha\theta}{(\theta-1)}\,w + w^2}$, $\alpha > 0, \theta > 2$
Weibull (α, β)	$-a + w \mp \sqrt{a^2 + 2a\beta\Gamma(1+\frac{1}{\alpha}) + \beta^2\Gamma\left(1+\frac{2}{\alpha}\right) - 2aw - 2\beta\Gamma(1+\frac{1}{\alpha})w + w^2}$, $\alpha > 0, \beta > 0$
LogNormal (μ, σ)	$-a + w \mp \sqrt{a^2 + 2ae^{\mu+\sigma^2/2} + e^{2\mu+2\sigma^2} - 2aw - 2e^{\mu+\sigma^2/2}w + w^2}$, $0 < \mu < \infty, \sigma > 0$
Burr (α, μ, λ)	$-a+w\mp\sqrt{a^2 + 2a\frac{\Gamma\left(\alpha-\frac{1}{\mu}\right)\Gamma\left(1+\frac{1}{\mu}\right)\lambda^{\frac{1}{\mu}}}{\Gamma\alpha} + \frac{\Gamma\left(\alpha-\frac{2}{\mu}\right)\Gamma\left(1+\frac{2}{\mu}\right)\lambda^{\frac{2}{\mu}}}{\Gamma\alpha} - 2aw - 2\frac{\Gamma\left(\alpha-\frac{1}{\mu}\right)\Gamma\left(1+\frac{1}{\mu}\right)\lambda^{\frac{1}{\mu}}}{\Gamma\alpha}w}$ $\alpha\mu > 2,\ \lambda > 0$

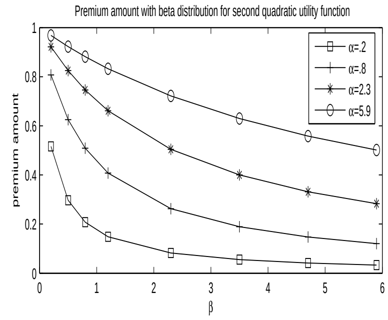

Figure 8. Beta Distribution $\mathrm{B}(\alpha, \beta)$

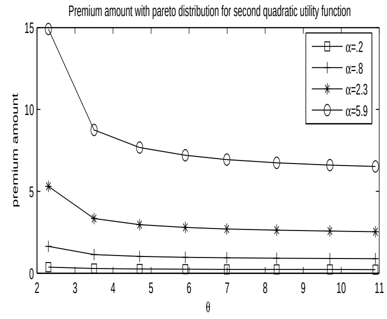

Figure 9. Pareto Distribution

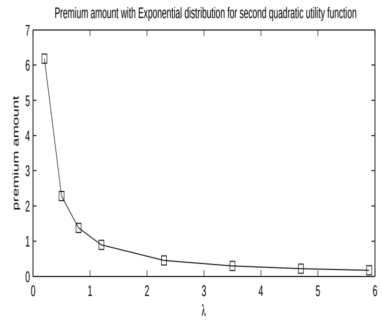

Figure 10. Exponential Distribution

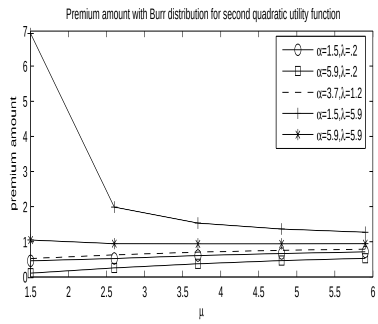

Figure 11. Burr Distribution

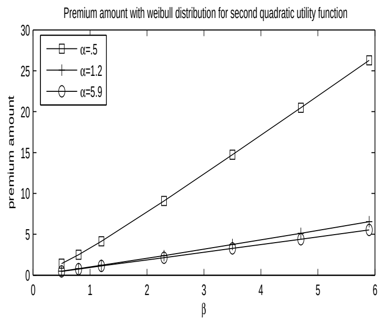

Figure 12. Weibull Distribution

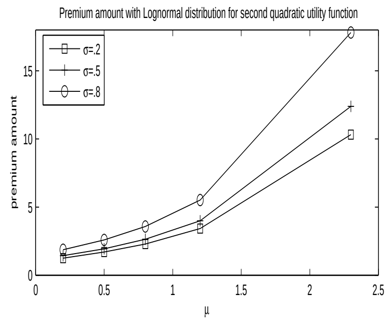

Figure 13. Lognormal Distribution

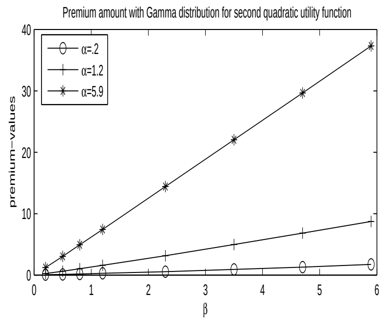

Figure 14. Gamma Distribution

The above figures lead to the conclusions that if insured has quadratic utility function, then

1. with Weibull, Lognormal and Gamma as loss distributions, the maximum premium increases as parametric values increase.This is beneficial for the insurer.
2. with Beta, Pareto and Exponential as loss distributions, the maximum premium declines with an increase in the parametric values. It is beneficial for the insured as he is willing to pay less premium but this causes loss to the insurer.
3. with Burr as loss distribution with $\alpha\mu > 2$
   - for $\alpha > \lambda$, the maximum premium increases,
   - for $\alpha \leq \lambda$, the maximum premium decreases and stabilises for higher values of μ.

4. Determination of maximum premium for Exponential utility function

For exponential utility function $u(w) = -\lambda e^{-\lambda w}$, using (1), the premium P_{max} is a solution of the equation

$$P_{max} = Log\left[M_X\left(\lambda\right)\right]/\lambda, \tag{4}$$

where $M_X(\lambda)$ is the moment generating function of the loss random variable X at λ (Bowers *et al.*, 1997). This is also known as Exponential

Premium Principle. The expressions for P_{max} values are compiled in the following table for Gamma, Beta and Exponential distribution.

Table 3. Maximum premium for Exponential utility function.

Distribution	P_{max}
Gamma (α, β)	$[Log(1-\beta\lambda)^{-\alpha}/\lambda], \alpha, \beta, \lambda > 0$
Beta (α, β)	$[Log_1 F_1(\alpha; \beta; \lambda)/\lambda], \alpha, \beta, \lambda > 0$ where ${}_1F_1(:;:;:)$ is the Generalised Hypergeometric function*
Exponential (α)	$\frac{1}{\lambda}\log\left(\frac{\alpha}{\alpha-\lambda}\right), \quad \alpha > \lambda$

*Generalised Hypergeometric Function is defined as

$$ {}_p^1F_q(a_1, a_2, a_3, \ldots, a_p; b_1, b_2, b_3, \ldots, b_q; x) = \sum_{k=0}^{\infty} \frac{\prod_{i=1}^{p} \ (a_i)_k \ x^k}{\prod_{i=1}^{q} (b_i)_k \ \ k!}, $$

where $(n)_k = \frac{\Gamma(n+k)}{\Gamma(n)}$ represents Pochhammer's symbol (Zwillinger, 2000).

Remarks 1. (i) As the moment generating functions do not exist in case of Pareto, Burr, Lognormal and Weibull distributions, (4) does not help in writing the expressions for P_{max}.

(ii) It is observed that for Exponential Distribution, the maximum premium values are negative or very close to zero. Hence it is practically not advisable to consider exponential distribution as loss distribution in combination with exponential utility function.

The following figures depict the trend of P_{max} for Gamma and Beta distributions for some combinations of parametric values.

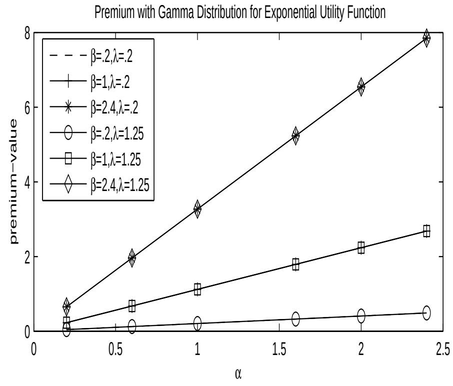

Figure 15. Gamma Distribution

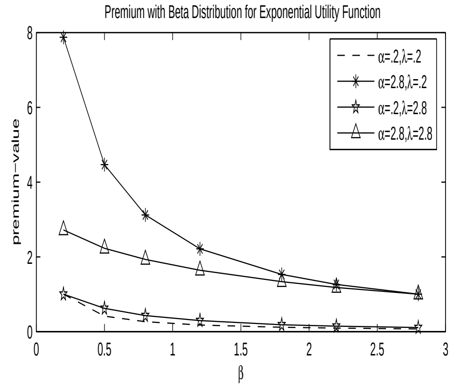

Figure 16. Beta Distribution

From the above figures, it is observed that

1. with Gamma as loss Distribution, the maximum premium increases as values of parameters increase, which is in the interest of the insurer,
2. with Beta Distribution, the maximum premium decreases as parameter values increase. This situation is beneficial for the insured.

5. Determination of maximum premium for fractional power utility function

The fractional power utility function is given by $u(w) = w^c, 0 < c < 1$ (Dickson, 2005).

Using (1), it is seen that the premium P_{max} is a solution of the equation

$$\frac{(c-1){P_{max}}^2}{2w} - P_{max} + E(X) - \left(\frac{c-1}{2w}\right) E(X^2) = 0. \tag{5}$$

Under different loss distributions and w=100, the expressions for P_{max} values are depicted in the following table:

Table 4. Maximum premium for fractional power utility function.

Distribution	P_{max}
Gamma (α, β)	$\frac{\left[w - \sqrt{w^2 - (c-1)\left(2w\alpha\beta - (c-1)(\alpha\beta^2 + \left(\alpha\beta)^2\right)\right)}\right]}{(c-1)}$, $\alpha > 0, \beta > 0;\ c \neq 1.$
Beta (α, β)	$\frac{\left[w - \sqrt{w^2 - (c-1)\left(2w\frac{\alpha}{(\alpha+\beta)} - (c-1)(\frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)})\right)}\right]}{(c-1)}$, $\alpha > 0, \beta > 0;\ c \neq 1$
Exponential (λ)	$\frac{\left[w - \sqrt{w^2 - (c-1)\left(2w\frac{1}{\lambda} - (c-1)(\frac{2}{\lambda^2})\right)}\right]}{(c-1)}$, $\lambda > 0$
Pareto (α, θ)	$\frac{\left[w - \sqrt{w^2 - (c-1)(2w\,\frac{\alpha\theta}{(\theta-1)} - (c-1)\,\frac{\alpha^2\theta}{(\theta-2)})}\right]}{(c-1)}$, $\theta > 2,\ \alpha > 0$
Weibull (α, β)	$\frac{\left[w - \sqrt{w^2 - (c-1)\left(2w\beta\Gamma(1+\frac{1}{\alpha}) - (c-1)\beta^2\Gamma(1+\frac{2}{\alpha})\right)}\right]}{(c-1)}$, $\alpha > 0,\ \ \beta > 0$
Lognormal (μ, σ)	$\frac{\left[w - \sqrt{w^2 - (c-1)\left(2we^{\mu+\sigma^2/2} - (c-1)e^{2\mu+2\sigma^2}\right)}\right]}{(c-1)}$, $0 < \mu < \infty,\ \sigma > 0$
Burr (α, μ, λ)	$\frac{\left[w - \sqrt{w^2 - (c-1)(2w\frac{\Gamma\left(\alpha-\frac{1}{\mu}\right)\Gamma\left(1+\frac{1}{\mu}\right)\lambda^{\frac{1}{\mu}}}{\Gamma\alpha} - (c-1)(\frac{\Gamma\left(\alpha-\frac{2}{\mu}\right)\Gamma\left(1+\frac{2}{\mu}\right)\lambda^{\frac{2}{\mu}}}{\Gamma\alpha})}\right]}{(c-1)}$, $\alpha\mu > 2,\ 0 < c < 1.$

For initial wealth $w = 100$, the following figures depict the trend of maximum premium P_{max} under different loss distributions.

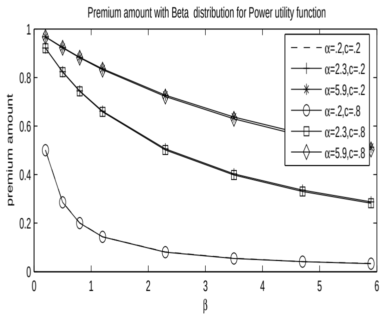

Figure 17. Beta Distribution

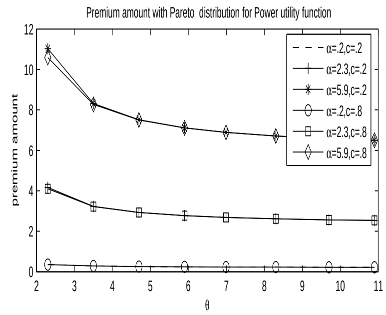

Figure 18. Pareto Distribution

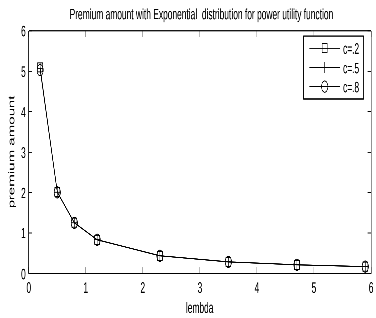

Figure 19. Exponential Distribution

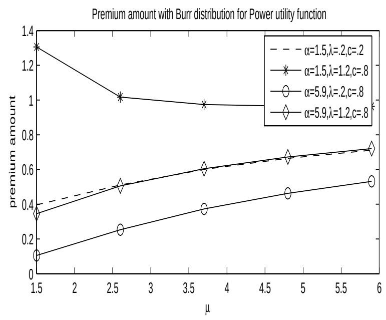

Figure 20. Burr Distribution

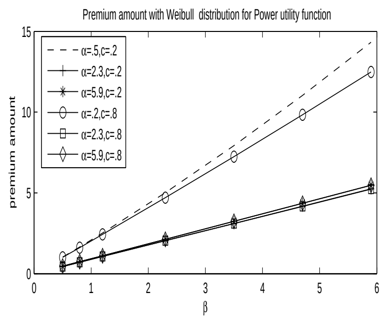

Figure 21. Weibull Distribution

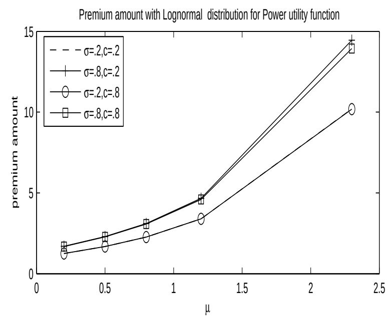

Figure 22. Lognormal Distribution

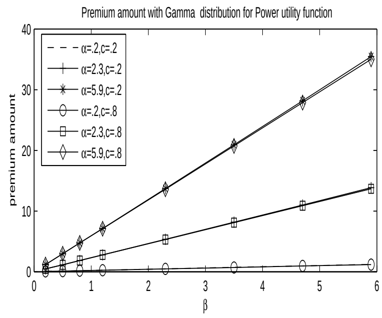

Figure 23. Gamma Distribution

From the above figures, we conclude that if insured has fractional power utility function

1. the maximum premium increases as parameters increase in case of loss distributions Weibull or Lognormal or Gamma. This is good from insurer's point of view,
2. the maximum premium declines as parametric values increase if insured has Beta or Exponential as loss distributions. This is beneficial for the insured as he has to be willing to pay less premium but causes loss to the insurer,
3. with loss distribution Burr, the maximum premium increases for any value of α, λ and c except when $1 < \alpha < 2$, $\lambda > 1$. It stabilises as μ increases,
4. with Pareto as loss distribution, P_{max} decreases as θ (shape parameter) increases. It is also observed that it is higher for larger α.

Now we compare the premium amounts for different utility functions and different loss distribution with similar values of the parameters.

6. Comparison of premium values for different utility functions under different loss distributions

The following table depicts values of P_{max} for Gamma, Pareto and Weibull distributions with same values of shape and scale parameters.

Table 5. Premium values

$U(w) = -(a-w)^2, w \leq a,\ w = 100,\ a = 105$		
Distribution	**Parameter values**	**Premium amount**
Gamma (α(shape), β(scale))	$\alpha = 2.3,\ \ \beta = .2$ $\alpha = 5.9,\ \beta = 5.9$	$P_{max} = 0.4684$ $P_{max} = 37.3109$
Pareto (α(scale), θ(shape))	$\theta = 2.3,\ \alpha = .2$ $\theta = 5.9,\ \alpha = 5.9$	$P_{max} = 0.3708$ $P_{max} = 7.1943$
Weibull (α(shape), β(scale))	$\alpha = 2.3,\ \beta = .2$ $\alpha = 5.9, \beta = 5.9$	$P_{max} = 0.1778$ $P_{max} = 5.5236$
$U(w) = w, w = 100$		
Distribution	**Parameter values**	**Premium amount**
Gamma (α(shape), β(scale))	$\alpha = 2.3,\ \beta = .2$ $\alpha = 5.9,\ \beta = 5.9$	$P_{max} = 0.46$ $P_{max} = 34.81$
Pareto (α(scale), θ(shape))	$\theta = 2.3,\ \alpha = .2$ $\theta = 5.9,\ \alpha = 5.9$	$P_{max} = 0.3538$ $P_{max} = 7.1040$
Weibull (α(shape), β(scale))	$\alpha = 2.3,\ \beta = .2$ $\alpha = 5.9,\ \beta = 5.9$	$P_{max} = 0.177183$ $P_{max} = 5.46844$
$U(w) = w^c,\ w = 100,\ c = .2$		
Distribution	**Parameter values**	**Premium amount**
Gamma (α(shape), β(scale))	$\alpha = 2.3,\ \beta = .2$ $\alpha = 5.9,\ \beta = 5.9$	$P_{max} = 0.2402$ $P_{max} = 35.4513$
Pareto (α(scale), θ(shape))	$\theta = 2.3,\ \alpha = .2$ $\theta = 5.9,\ \alpha = 5.9$	$P_{max} = 0.3546$ $P_{max} = 7.1124$
Weibull (α(shape), β(scale))	$\alpha = 2.3,\ \beta = .2$ $\alpha = 5.9,\ \beta = 5.9$	$P_{max} = 0.1772$ $P_{max} = 5.4729$

The values in the above table lead to the conclusion that for quadratic and linear utility functions, P_{max} is highest for Gamma distribution for different combinations of parameter values.

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