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APPLICATION OF HLM TO DATA WITH MULTILEVEL STRUCTURE

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Abstract

Many data sets analyzed in human and social sciences have a multilevel or hierarchical structure. By hierarchy we mean that units of a certain level (also referred micro units) are grouped into, or nested within, higher level (or macro) units. In these cases, the units within a cluster tend to be more different than units from other clusters, i.e., they are correlated. Thus, unlike in the classical setting where there exists a single source of variation between observational units, the heterogeneity between clusters introduces an additional source of variation and complicates the analysis.

Collecting data on Educational Research often does not follow the principles of simple random sample, suspected by classical regression, but rather a sample by nested clusters. Selected to students and also the contextual units to which they belong such as classes, courses, schools, neighborhoods or regions, and so forth.

Using classical regression bias is produced in the typical error of measurement and an increased likelihood of committing errors of inference. The hierarchical linear or multilevel models are most suitable because they consider the hierarchical relationships and also provide estimates on the contextual variability of regression coefficients.

In practice, often the data structures are not hierarchical, are more complex structures such as cross-classification (level 2 or macro). For example, students (level 1 or micro) to attend different courses at a school while in other schools there are students who attend the same courses.

Two examples of application to academic achievement of students are presented. First, a model of cross-classification of level 2 is used. Second, a hierarchical model of two levels (students and schools) is presented, taking into account the different areas of science - scientific-humanistic courses and technology courses.

Keywords: hierarchical linear model, multilevel model, cross-classification models, academic achievement.

2010 Mathematics Subject Classification: 62–07, 62P25.

1. Introduction

The single-level analysis that ignore the multilevel (hierarchical) structure of the data can provide misleading results. Hierarchical linear models (HLM), or multilevel models, take the multilevel structure of educational data into account, and they provide a conceptual and statistical mechanism for investigating and drawing conclusions regarding the influence of phenomena at different levels of analysis.

These models can simultaneously examine effects of both individual and group level variables on an individual level outcome. Moreover, the correlated errors and nonzero ICC (*intra-class correlation* – is a basic measure for the degree of dependency in clustered observations) inherent in grouped data are appropriately incorporated in HLM, giving accurate standard errors estimates and inferences.

HLM have originally been developed in educational and social research where observations are often made on different levels simultaneously (such as students, classes, schools, and so forth) (Richter, 2006) [10].

Hierarchical linear models (Bryk and Raudenbush, 1992 [2]; Snijders and Bosker, 1999) [11], also known as *multilevel models* (Goldstein, 1995) [3], *multilevel regression models* (Hox, 2002) [4], or *random coefficient models* (Kreft and de Leeuw, 1998) [5] are all forms of multi-level modeling and are, essentially, equivalent to one another. We consider two-level hierarchical data structures and follow the notation of Bryk and Raudenbush (1992) [2].

In our work we have data in J groups or contexts (schools), and a different number of individuals (students) n_j in each group. The data do not have to be necessarily balanced (it is not necessary that $n_j = n_k$ to $n \neq k$). On the student level (lowest) we have the dependent variable Y_{ij} and the explanatory variable X_{ij}, and on the school level we have the explanatory variable W_j.

Thus, in the two-level hierarchical models, we can have separate level-1 regression equations at each of the level 2 units. The level-1 or within school model can be represented as:

$$Y_{ij} = \beta_{0j} + \beta_{1j}\ X_{ij} + e_{ij},$$

where Y_{ij} is the outcome for the ith student in the jth school; X_{ij} is the explanatory variable for the ith student in the jth school; β_{0j} is the intercept for the jth school; β_{1j} is the slope for the jth school; and e_{ij} is the random error to the ith student in the jth school from its school's predicted line. The subscripts for the β coefficients in this equation indicate that they can differ for each school j.

Intercepts and slopes are modeled by explanatory variables in the level-2 or between school models as:

$$\beta_{0j} = \gamma_{00} + \gamma_{01}\ W_j + u_{0j},$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}\ W_j + u_{1j},$$

where γ_{00} is the estimated intercept when W_j is equal to zero; u_{0j} is the random error to the jth school from the average intercept; γ_{10} is the estimated slope when W_j is equal to zero; and u_{1j} is the random error to the jth school from the average slope. The γ_{01} and γ_{11} are the regression coefficients associated with the effects of the explanatory's school level on the student level structural relationships. Substitution of these equations (level-2 models) in level-1 model gives:

$$Y_{ij} = \gamma_{00} + \gamma_{10}\ X_{ij} + \gamma_{01}\ W_j + \gamma_{11}\ W_j\ \ X_{ij} + u_{0j} + u_{1j}\ X_{ij} + e_{ij}.$$

X and W variables can be modeled in their original, untransformed metric or centered (about respective grand means, or X about respective group means) (Sullivan *et al.*, 1999) [12].

The specification of error terms at both the student (e) and school (u) levels allows HLM's to appropriately model the error in grouped data (i.e., non zero ICC).

The HLM's assumptions are extensions of the linear modeling restrictions required for single level OLS regression (Bryk and Raudenbush, 1992 [2]; Snijders and Bosker, 1999) [11]. The mathematical expressions of the assumptions can be found, for example, in Sullivan *et al.* (1999) [12].

In practice, often the "multilevel data do not always follow a strictly hierarchical structure (Raudenbush and Bryk, 2002) [9]", are more complex structures such as, cross-classified models – for instance, students to attend different courses at a school while in other schools there are students who attend the same courses (Raudenbush and Bryk, 2002) [9]. Therefore school is not nested within the course and the course is not nested within school: instead, we have cross-classified structure (Rasbash *et al.*, 2004) [8].

A simple model in this context can be written as (we follow Rasbash *et al.*, 2004) [8]:

$$Y_{i(jk)} = \beta_0 + v_{0k} + u_{0jk} + e_{i(jk)}$$

with

$$v_{0k} \sim N\left(0,\ \sigma^2_{v0}\right);\ u_{0jk} \sim N\left(0,\ \sigma^2_{u0}\right);\ e_{i(jk)} \sim N\left(0,\ \sigma^2_{e}\right),$$

where the academic achievement score $Y_{i(jk)}$ of the ith student from the (jk)th course/school combination is modeled by the overall mean β_0, together with a random error v_{0k} due to school k, a random error u_{0jk} due to course j, and an individual-level random error $e_{i(jk)}$. In this model we have students at level-1 and courses and schools are cross-classified at level-2. For details, see Rasbash and Browne (2008) [7].

More detailed discussion of multilevel or HLM's procedures can be found in Bryk and Raudenbush (1992) [2], Goldstein (1995) [3], Kreft and de Leeuw (1998) [5], Snijders and Bosker (1999) [11], Hox (2002) [4], and Raudenbush and Bryk (2002) [9].

2. Application, results and analysis

The data are withdrawn from a list of questions, applied to the 10th grade high school students in 2004/2005 considering three subjects of the scientific areas. In order to show that HLM are appropriate to identify relevant factors of the students' performance, we aim to identify if there are relevant differences on average students' performance between schools and courses; which explanatory variables at different levels affect the output variable and how much variability we must have at each output level; and if the students' distribution by school is random.

Based on Valente and Oliveira (2006, 2009) [15] [13] one cross-classified model of level 2 (classes and schools), and two HLM's of 2 level (students and schools), taking into account the different areas of science – scientific-humanistic courses and technological courses.

Throughout this work, we use the package *MlwiN* 2.23 [6], developed and described by Rasbash *et al.* (2004) [8] and Browne (2004) [1]. More detailed description of selected variables that are used in the construction of the different intermediate models and final models can be found in Valente and Oliveira (2007, 2009) [14] [13].

In analysis of HLM's a preliminary study with the explanatory variables is made, in order to verify both, contribution and significance, in future models (Valente and Oliveira, 2007; 2009) [14] [13].

Table 1. Cross-Classified Models

	Unconditional Model	**Variance Components Model**	**Variance Components Model** (with CURSO)
Parameters	**Estimate** (s.e)	**Estimate** (s.e)	**Estimate** (s.e)
FIXED			
Intercept	0.008 (0.027)	-0.048(0.066)	-0.563(0.086)***
CURSO			0.732(0.085)***
RANDOM			
Level 2: Schools – Intercept		0.023(0.024)	0.042(0.025)*
Level 2: Classes – Intercept		0.203(0.044)***	0.075(0.024)***
Level 1: Students	0.976(0.037)***	0.756(0.030)***	0.755(0.029)***
Deviance(MCMC)	3899.88	3546.14	3544.92
Number of valid data	1387	1387	1387
DIC (pD)	3901.89(2.01)	3614.63 (68.49)	3601.24 (56.32)

*** Significant to $p \leq 0.001$ ** Significant to $p \leq 0.01$ * Significant to $p \leq 0.05$

In Table 1 we have models of two levels where classes and schools are cross-classified at level 2. In variance components model we can see that classes are actually more important in predicting the academic achievement score than schools. The schools only explains $0.023/(0.023 + 0.203 + 0.756) \cdot 100\% = 2.34\%$ of variation – the coefficient is not significant, while the classes explain $0.203/(0.023 + 0.203 + 0.756) \cdot 100\% = 20.67\%$. The DIC diagnostic shows that this model is an improvement with a reduction in DIC value of over $280(3901.89 - 3614.24 = 287.26)$. This indicates that within-school differences (between students) are far larger than between-school differences (and even than between classes).

Adding the explanatory variable CURSO (type of course: Science/ Humanities or Technology) has the effect of reducing the variability value between classes around 63.05% (σ^2_{u0} change from 0.203 to 0.075) meanwhile the variability value between pupils is null. This variable introduces differences between schools so that the coefficient is now significant although slightly (σ^2_{v0} change from 0.023(0.024) to 0.042(0.025)).

In our last researches we found that the students of sciences and humanities perform better than students of technological courses (Tables 1 and 2), therefore we decided to further study to obtain comparisons between courses in different subjects areas and whys.

We choose to aggregate some explanatory variables in intermediate models to test its significance while a group. First, the variance components models are presented, i.e., without explanatory variables in Table 2.

Table 2. Cross-Classified Model – Random Intercept

	Technological Courses	**Scientific-humanistic Courses**
Parameters	**Estimate** (s.e)	**Estimate** (s.e)
FIXED		
Intercept	-0.550(0.078)***	0.167(0.068)**
RANDOM		
Level 2: Schools – Intercept	0.019(0.024)	0.052(0.030)*
Level 2: Classes – Intercept	0.076(0.041)*	0.071(0.030)**
Level 1: Students	0.579(0.045)***	0.817(0.038)***
Deviance(MCMC)	821.42	2708.86
Number of valid data	359	1028
DIC (pD)	840.18(18.75)	2748.35(39.49)

*** Significant to $p \leq 0.001$ ** Significant to $p \leq 0.01$ * Significant to $p \leq 0.05$

<u>Technologic Courses</u>: 0.579/0.674 = 85.9% is the proportion of variance within classes; 0.076/0.674 = 11.3% is the proportion of variance among classes within schools; and 0.019/0.674 = 2.8% is the proportion of variance among schools, where 0.674 = (0.019+0.076+0.579).
<u>Scientific-humanistic courses</u>: 0.817/0.940 = 86.9% is the proportion of variance within classes; 0.052/0.940 = 7.6% is the proportion of variance among classes within schools; and 0.052/0.940 = 5.5% is the proportion of variance among schools, where 0.940 = (0.052+0.071+0.817).

The results for classes and schools are a quite low, compared to other results of educational research – values between 0.05 and 0.20 are common (Snijders & Bosker, 1999) [11]. This indicates that the grouping to schools (classes) leads to a low similarity between the results of different students in the same school (class), although within-school (within-class) differences are far larger than between-school (between-class) differences.

Estimated residuals, at any level, can be used to check model assumptions. One such assumption is that the residuals at each level follow Normal distributions. This assumption may be checked using a Normal probability plot, in which the ranked residuals are plotted against corresponding points on a Normal distribution curve. If the Normality assumption is valid, the points on a Normal plot should lie approximately on a straight line (Rasbash *et al.* 2004) [8], as we can see in Figure 1.

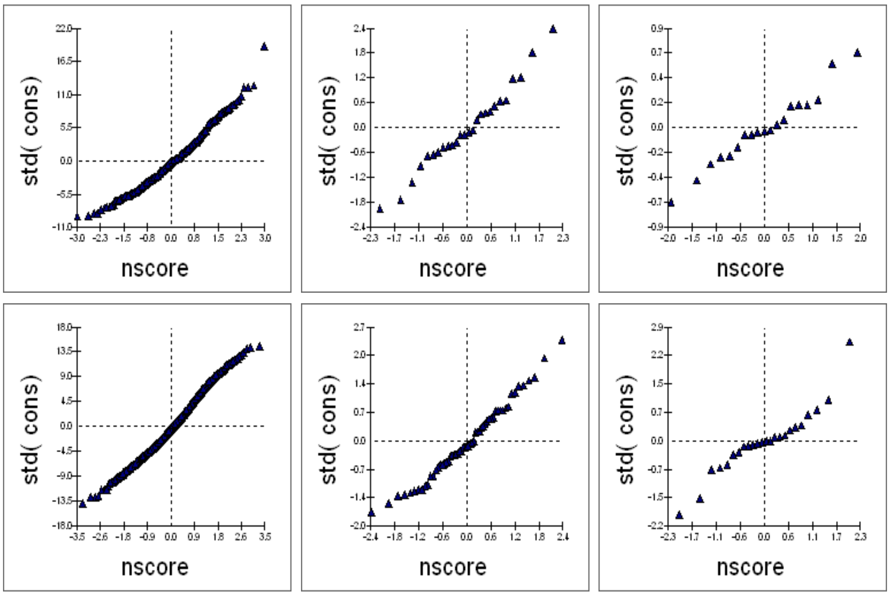

Figure 1. Plot of student (left), class (middle) and school (right) residuals for cross-classified at level 2 model. Technological courses (top); Scientific-humanistic courses (bottom).

The plots looks fairly linear, which suggests that the assumption of Normality is reasonable.

In Table 3, it seems there is no evident variability both between schools as between classes (technologic and scientific-humanistic courses). These variability's are practically explained by this variable (CURSO) – some coefficients are not significant anymore. The variability between classes is explained in 81.6% (5.6%) and between schools is 47.4% worse (9.6% worse) for technologic courses (scientific-humanistic courses).

Table 3. Base – Cross-Classified Model

Parameters	Technological Courses Estimate (s.e)	Scientific-humanistic Courses Estimate (s.e)
FIXED		
Intercept	-0.325(0.117)***	0.332(0.152)*
C_TEC_ASoc	0.296(0.215)	
C_TEC_Desp	-0.229(0.179)	
C_TEC_Elec	-0.535(0.169)***	
C_TEC_Inf	-0.430(0.144)**	
C_TEC_Mark	-0.258(0.352)	
C_TEC_Mult	-0.035(0.334)	
C_CHUM_CSE		-0.321(0.172)*
C_CHUM_CSH		-0.174(0.175)
C_CHUM_CT		-0.137(0.159)
RANDOM		
Level 2: Schools – Intercept	0.028(0.028)	0.057(0.034)*
Level 2: Classes – Intercept	0.014(0.018)	0.067(0.028)**
Level 1: Students	0.576(0.045)***	0.818(0.038)***
Deviance(MCMC)	820.09	2708.53
Number of valid data	359	1028
DIC (pD)	836.55(16.46)	2748.86(40.32)

*** Significant to $p \leq 0.001$ ** Significant to $p \leq 0.01$ * Significant to $p \leq 0.05$
C_TEC_Adm and C_CHUM_AV are the reference categories for Technological Courses and SH-Courses, respectively.

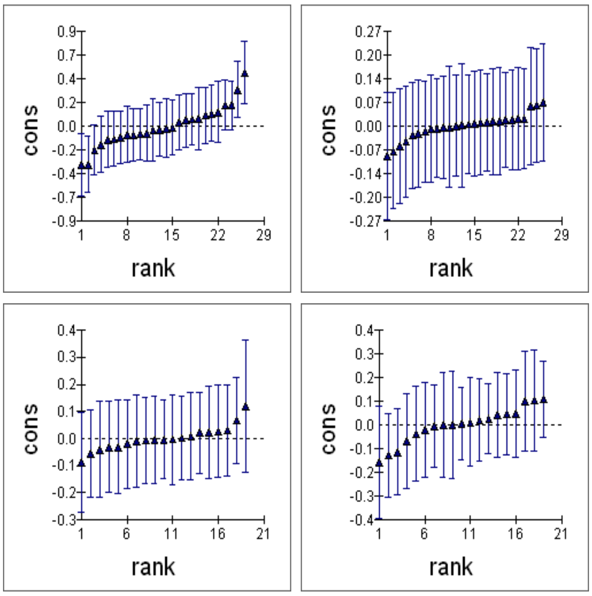

Figure 2.1. Caterpillar plot of level 2 residuals for Technologic courses. Classes (top) and schools (bottom). Cross-classified model – random intercepts (left) and Base cross-classified model (right).

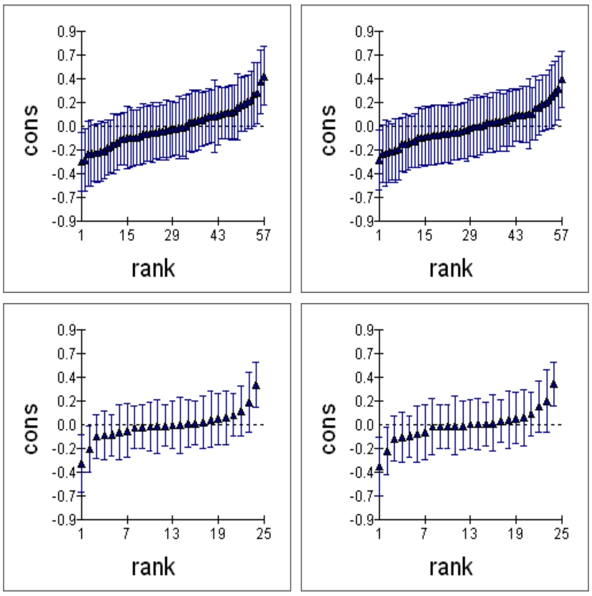

Figure 2.2. Caterpillar plot of level 2 residuals for Scientific-humanistic courses. Classes (top) and schools (bottom). Cross-classified model – random intercepts (left) and Base cross-classified model (right).

The diagnostic DIC value for models of both courses don't suggests an improvement. So, we choose to use two-level hierarchical linear models – students and schools (tables 4 to 6) without interactions.

Looking at the Figures 2.1 and 2.2, we can see that the confidence intervals around the residuals do not overlap zero for both classes and schools – model of Table 2, and for both classes and schools – model of Table 3.

Also now, we have chosen to aggregate some exploratory variables in intermediate models to test its significance while a group.

From Table 4, we can see that $0.063/0.672 = 9.4\%$ is the proportion of variance between schools and $0.609/0.672 = 90.6\%$ is the proportion of variance between students for Technological Courses, where $0.672 = (0.063+0.609)$. We have a proportion of variance of $0.074/0.934 = 7.9\%$ and $0.860/0.934 = 92.1\%$, between schools and between students, respectively, for SH-Courses.

Table 4

	Uncondicional Model		Model – Random Intercept	
	Technological Courses	Scientific-humanistic Courses	Technological Courses	Scientific-humanistic Courses
Parameters	**Estimate** (s.e)	**Estimate** (s.e)	**Estimate** (s.e)	**Estimate** (s.e)
FIXED				
Intercept	-0.534(0.043)***	0.197(0.030)***	-0.536(0.072)***	0.164(0.064)**
RANDOM				
Level 2: Schools – Intercept			0.063(0.035)*	0.074(0.031)**
Level 1: Students	0.664(0.050)***	0.994(0.042)***	0.609(0.048)***	0.860(0.039)***
Deviance(MCMC)	873.85	2854.86	839.88	2759.93
Number of valid data	359	1028	359	1028
DIC (pD)	875.82 (1.98)	2858.86 (2.00)	852.84 (12.96)	2778.57 (18.64)

*** Significant to $p \leq 0.001$ ** Significant to $p \leq 0.01$ * Significant to $p \leq 0.05$

Table 5. Base Model

	Technological Courses	Scientific-humanistic Courses
Parameters	**Estimate** (s.e)	**Estimate** (s.e)
FIXED		
Intercept	-0.301(0.114)**	0.314(0.112)**
C_TEC_ASoc	0.269(0.204)	
C_TEC_Desp	-0.250(0.167)	
C_TEC_Elec	-0.564(0.160)***	
C_TEC_Inf	-0.456(0.128)***	
C_TEC_Mark	-0.285(0.349)	
C_TEC_Mult	-0.050(0.327)	
C_CHUM_CSE		-0.312(0.116)**
C_CHUM_CSH		-0.156(0.118)
C_CHUM_CT		-0.124(0.106)
RANDOM		
Level 2: Schools – Intercept	0.039(0.033)	0.075(0.032)**
Level 1: Students	0.577(0.045)***	0.854(0.039)***
Deviance(MCMC)	820.36	2753.87
Number of valid data	359	1028
DIC (pD)	835.90(15.54)	2775.49(21.62)

*** Significant to $p \leq 0.001$ ** Significant to $p \leq 0.01$ * Significant to $p \leq 0.05$
C_TEC_Adm and C_CHUM_AV are the reference categories for Technological Courses and SH-Courses, respectively.

The base model (Table 5) is formed by categorical variable CURSO (the course chosen by the student). This model explains 38.1% of the existing variability between schools and 5.3% of the variability between students – technological courses. The model doesn't improve for scientific-humanistic courses.

In Table 6.1, the model A deals with students characteristics. The students with age-grade imbalance (D_IDADE) have a tendency to score poorly. Male students (SEXO) perform poorer than female students – SH courses. The model contributes with 14.7% and 4.9% for the explanation of the difference between schools and between students, respectively – SH courses. This suggests that students are not distributed by schools in a random way.

Model B (Table 6.1) is formed by variables representing students' attitudes. This is a good example of the students' practice. The coefficients of the variables of the scientific-humanistic courses are statistically very significant compared with the coefficients of technological courses.

Table 6.1. Results of two-level HLM – Models A and B

	Model A		Model B	
	Technological Courses	Scientific-humanistic Courses	Technological Courses	Scientific-humanistic Courses
Parameters	**Estimate** (s.e)	**Estimate** (s.e)	**Estimate** (s.e)	**Estimate** (s.e)
FIXED				
Intercept	-0.155(0.119)	0.520(0.113)***	-0.894(0.243)***	-0.699(0.199)***
C_TEC_ASoc	0.311(0.196)		0.269(0.207)	
C_TEC_Desp	-0.268(0.160)*		-0.261(0.158)*	
C_TEC_Elec	-0.569(0.167)***		-0.565(0.157)***	
C_TEC_Inf	-0.488(0.133)***		-0.445(0.120)***	
C_TEC_Mark	-0.272(0.350)		-0.238(0.354)	
C_TEC_Mult	-0.108(0.318)		-0.113(0.320)	
C_CHUM_CSE		-0.323(0.114)**		-0.370(0.105)***
C_CHUM_CSH		-0.184(0.117)		-0.168(0.107)
C_CHUM_CT		-0.218(0.106)*		-0.270(0.096)**
SEXO	0.014(0.102)	-0.066(0.059)		
NEE	-0.372(0.240)	-0.318(0.210)		
D_IDADE	-0.175(0.047)***	-0.364(0.054)***		
A_EMP			0.695(0.157)***	0.699 (0.107)***
A_PART			0.214(0.147)	0.773(0.090)***
A_DIST			-0.211(0.143)	-0.446(0.098)***
A_ASSID			0.223(0.174)	0.446(0.145)**
AJU_TPC			-0.188(0.110)*	-0.233(0.080)**
RANDOM				
Level 2: Schools – Intercept	0.037(0.032)	0.064(0.028)**	0.043(0.035)	0.055(0.025)*
Level 1: Students	0.555(0.043)***	0.812(0.036)***	0.511(0.040)***	0.680(0.032)***
Deviance(MCMC)	806.08	2701.66	763.78	2461.17
Number of valid data	359	1028	353	1004
DIC (pD)	824.50(18.42)	2725.90(24.24)	785.04(21.25)	2487.56(26.39)

*** Significant to $p \leq 0.001$ ** Significant to $p \leq 0.01$ * Significant to $p \leq 0.05$
C_TEC_Adm and C_CHUM_AV are the reference categories for Technological Courses and SH-Courses, respectively.

Model C (Table 6.2) is composed of variables that show the attitudes and expectations of the students and families towards the schools and the studies. It is quite significant and about 48% of the variability between schools (SH courses) is explained by these variables and, more than 14% of the variability between students is explained by this model. The model contributes with more than 12% for the explanation of the difference between students and about 28% worse for the explanation of the difference between schools – technological courses. This suggests that students are not distributed by schools in a random way.

Table 6.2. Results of two-level HLM – Models C and D

	Model C		Model D	
	Technological Courses	Scientific-humanistic Courses	Technological Courses	Scientific-humanistic Courses
Parameters	**Estimate** (s.e)	**Estimate** (s.e)	**Estimate** (s.e)	**Estimate** (s.e)
FIXED				
Intercept	-0.319(0.173)*	-0.242(0.136)*	-0.691(0.195)***	-0.689(0.167)***
C_TEC_ASoc	0.351(0.214)*		0.324(0.203)*	
C_TEC_Desp	-0.234(0.158)		-0.292(0.164)	
C_TEC_Elec	-0.576(0.153)***		-0.568(0.157)***	
C_TEC_Inf	-0.555(0.124)***		-0.504(0.131)***	
C_TEC_Mark	-0.251(0.365)		-0.382(0.358)	
C_TEC_Mult	-0.067(0.323)		-0.084(0.329)	
C_CHUM_CSE		-0.442(0.107)***		-0.328(0.114)**
C_CHUM_CSH		-0.244(0.110)*		-0.130(0.115)
C_CHUM_CT		-0.319(0.100)***		-0.169(0.105)
REP_ANT	-0.316(0.085)***	0.533 (0.081)***		
UNIVERS	0.435(0.092)***	0.557(0.064)***		
IMP_ESC	0.193(0.141)	0.305(0.099)***		
MAE_EMP	-0.141(0.088)			
PARENTAL		0.204(0.066)***		
MHAB_LIT			0.008(0.011)	0.042 (0.007)***
N_ASSOAL			0.087(0.047)*	0.103(0.037)**
INTERNET			0.133(0.091)	0.117(0.073)**
TEL_FIXO				0.207(0.078)
RANDOM				
Level 2: Schools – Intercept	0.050(0.037)	0.039(0.019)*	0.041(0.036)	0.053(0.024)*
Level 1: Students	0.504(0.037)***	0.733(0.033)***	0.571(0.045)***	0.797(0.036)***
Deviance(MCMC)	759.14	2569.27	798.36	2635.27
Number of valid data	353	1017	351	1010
DIC (pD)	779.73(20.59)	2592.61(23.24)	816.82(18.45)	2659.57(24.31)

*** Significant to $p \leq 0.001$ ** Significant to $p \leq 0.01$ * Significant to $p \leq 0.05$
C_TEC_Adm and C_CHUM_AV are the reference categories for Technological Courses and SH-Courses, respectively.

In the model D (Table 6.2) there are variables related to the family characteristics, MHAB_LIT (the best academic achievement of the parents) and variables related to possessions and services available at student's home. That, in a certain extent, is related to the socio-economical level of the family. The model is very significant for SH courses: the variability between schools is explained about 29% and between students is 6.7%. This also suggests that students are not distributed by schools in a random way.

Model E (Table 6.3) is composed by variables concerning the location of schools – URBANA and SUB_URB (areas) – and by aggregate variables concerning the age of students – MDIDA_T and MDIDA_E (average D_IDADE within classes and schools, respectively). There is a sharp contrast between courses, either on the location of schools either on the average age of students in classes and schools. This model is quite significant and the variability between schools is practically explained by these variables (81% for technological courses and 44% for SH courses), but the explained variability between students is practically null (5.9% and 0.2%, for technological courses and SH courses, respectively). This perhaps contributes to explain the considerable difference between schools.

Table 6.3. Results of two-level HLM – Model E

	Technological Courses	**Scientific-humanistic Courses**	**Scientific-humanistic Courses**–random slopes
Parameters	**Estimate** (s.e)	**Estimate** (s.e)	**Estimate** (s.e)
FIXED			
Intercept	-0.697(0.214)***	0.359(0.151)**	0.362(0.160)**
C_TEC_ASoc	0.163(0.172)		
C_TEC_Desp	-0.194(0.152)		
C_TEC_Elec	-0.461(0.145)***		
C_TEC_Inf	-0.502(0.116)***		
C_TEC_Mark	-0.539(0.318)*		
C_TEC_Mult	0.211(0.307)		
C_CHUM_CSE		-0.304(0.115)**	-0.169(0.125)
C_CHUM_CSH		-0.161(0.114)	-0.095(0.127)
C_CHUM_CT		-0.214(0.117)*	-0.171(0.123)
URBANA		0.334(0.105)	0.209(0.089)**
SUB_URB	-0.297(0.144)*		
MDIDA_T		-0.350(0.167)*	-0.514(0.255)*
MDIDA_E	0.866(0.358)**		
RANDOM			
Level 2: Schools – Intercept	0.012(0.016)	0.035(0.020)*	0.100(0.050)*
Level 2: Schools – Interaction			-0.264(0.125)*
Level 2: Schools – Slope			0.754(0.345)*
Level 1: Students	0.573(0.045)***	0.852(0.038)***	0.833(0.037)***
Deviance(MCMC)	818.36	2752.82	2727.26
Number of valid data	359	1028	1028
DIC (pD)	831.14(12.78)	2772.09(19.27)	2750.93(23.68)

*** Significant to $p \leq 0.001$ ** Significant to $p \leq 0.01$ * Significant to $p \leq 0.05$
C_TEC_Adm is the reference category and C_CHUM_AV is the reference category for Technological Courses.

The final models (Models C and E – SH courses and, Model E – technological courses, mainly) decrease the variability between schools and a little between students. Comparing the final models with the unconditional model (or even the base model) we can observe that the change in the deviance value is very highly significant, confirming the better fit of the "more elaborated models" to the data.

Adding the explanatory variables has the effect of reducing the DIC diagnostic, suggesting that the differences between schools and between students have been partly explained by the additional variables – some of the coefficients (for schools) are not significant anymore.

3. CONCLUSIONS

In our research, we found some evidences:

The students with age-grade imbalance have a tendency to score poorly.

Students of sciences and humanities perform better than students of technological studies.

Students (SH courses) with parental family have a better performance. In what concerns to students from schools localized in "rural" region – the reference region:

- students from "urban" schools (SH courses) have better performance than the "rural" ones;
- students from "suburban" schools (technological courses) have poorer performance than the "rural" ones.

The existence of variability between schools is more difficult to explain in scientific-humanistic courses.

It was found that students in technological courses show:

- higher "age-grade imbalance"; so,
- . . . higher age of attendance of 10th grade;
- higher repetition in years prior to the 10th;
- to attend, above all, schools localizated in "suburban" region;
- "best academic achievement of the parents" are poorer and have lower influence;
- seem to have a greater "uniformity" among themselves.

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