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MARKET EFFICIENCY AND NON-LINEAR DEPENDENCE IN THE CZECH CROWN/US DOLLAR FOREIGN EXCHANGE MARKET

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Abstract

We examine the Czech Crown/US Dollar exchange rate for evidence of market efficiency during the period May, 1997, to September, 1998. The Czech Crown was floated on the world's foreign exchange markets in May, 1997, and it is of interest to examine the behaviour of this new market. We show that this foreign exchange market satisfied the criteria for weak form efficiency during the first part of the period under investigation but there is evidence of non-linear dependence during the second part of the period. This is successfully modelled using a $GARCH - M(1,1)$ representation.

Keywords: foreign exchange markets, market efficiency, time series analysis, GARCH models.

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1. Introduction

The dynamic behaviour of foreign exchange markets has long been a topic of interest to financial economists and there is now a vast literature on the subject. The majority of studies have examined the properties of foreign exchange rates between the major world economies (see, for example, Hsieh, 1988, and Pan, Liu and Bastin, 1996, and the numerous references contained therein). The purpose of this paper is to examine the dynamic behaviour of an 'emerging market' currency: the Czech koruna (crown). As a currency of only the Czech Republic[1], the crown was introduced on 9^{th} Feb 1993 and pegged to a basket consisting of the German Mark (65%) and the US dollar (35%). On 1st October 1995 the government made the crown freely *convertible*, but still pegged to the Mark/Dollar basket; on 26^{th} May 1997 the ties to the basket of currencies were effectively ended and the crown was allowed to float freely in the market place. We are not aware of any previous studies in the literature which are concerned with the Czech crown since its flotation and this, perhaps, lends interest to the present paper. The plan of the rest of the paper is as follows. Section 2 discusses time series modelling of exchange rates and our interest is focused on short-term dynamic models using high frequency data. We introduce the concept of foreign exchange market efficiency and its modelling implications, and explain how this may be tested for. In the light of the results of Section 2, Section 3 suggests a suitable time series model for the data and carries out the estimation and diagnostic testing. Our preferred model is a variant of the generalised auto-regressive conditional heteroscedasticity (*GARCH*) class of models (Bollerslev, 1987) and provides a satisfactory fit to the data. Finally, Section 4 offers some concluding remarks.

2. Foreign exchange market efficiency

The concept of market efficiency provides a convenient starting point for dynamic modelling of exchange rates using high-frequency data. In its weak form, the efficient market hypothesis (*EMH*) states that the successive day-to-day changes in the logarithm of the exchange rate are independently and identically distributed (*IID*) random variables. In other words, the exchange rate is expected to follow a random walk (Fama, 1970, pp. 383–90). The main implication is that future exchange rate changes

[1]Prior to February 1993, the crown was also the currency of Slovakia.

cannot be predicted from past changes because the information set conditioning the current exchange rate contains information on all past exchange rates. This means that if X_t is the exchange rate at time t, then:

$$\log(X_t) = \log(X_{t-1}) + u_t \tag{1}$$

where u_t is a sequence of zero-mean IID random variables. It follows that the exchange rate series, X_t, will be an $I(1)$ series[2] and the ut series will be a stationary, $I(0)$, series. To ascertain the order of integration of the exchange rate series, we carry out tests for unit roots using augmented Dickey-Fuller (ADF) statistics (Dickey and Fuller, 1979). The data were supplied by Datastream and consist of the daily closing rates for the Czech Crown/US Dollar exchange rate from 27^{th} May 1997 to 16^{th} September 1998; a total of 342 data points. The data are illustrated in Figure 1 and the results of the unit root tests are given in Table 1 below.

Figure 1. Czech Crown/US Dollar exchange rate.

[2]i.e. will need to be differenced once to render it stationary.

1: Unit root (ADF) Test Statistics

(i)	**For the exchange rate:**	-1.8253
(ii)	**For the first differences:**	-9.0920
	1% Critical Value*	-3.4517
	5% Critical Value	-2.8703
	10% Critical Value	-2.5714

* MacKinnon (1991) critical values for rejection of hypothesis of a unit root.

To test whether ut is an IID series, we calculate BDS statistics (Brock *et al.*, 1987). The BDS statistic provides a statistical test of IID within a time series, and is based upon the correlation dimension (Grassberger and Procaccia, 1983). Brock *et al.*, (1987) show that for a time series which is IID, the BDS statistic is asymptotically distributed as $N(0,1)$. Let:

$$W_m(\varepsilon) = \frac{\sqrt{n}\,[C_m(\varepsilon) - {C_1}^m(\varepsilon)]}{\sigma_m(\varepsilon)} \tag{2}$$

where $C_m(\varepsilon)$ represents the fraction of all m-tuples in the series which are 'close' to (within ε of) each other and $\sigma_m(\varepsilon)$ is an estimate of the standard deviation. $Wm(\varepsilon)$ is the BDS statistic and provides a formal test of the IID assumption. Usually, ε takes values ranging from 0.5 to 2 standard deviations of the data to be tested and m, the embedding dimension, is given integer values between about 2 and 8. We should, however, at this point explain that:

(a) We split the sample at 8^{th} January, 1998, because of the apparent structural break at that time[3] (see Figure 1 above). We calculate separate sets of BDS statistics for the period ending 8th January 1998 (period 1) and the period commencing 9^{th} January 1998 (period 2).

(b) The BDS statistics were calculated for the series $u_t = \ln(X_t/X_{t-1})$ because it is ut which we intend to develop a model for in the following

[3]We note that this break comes immediately after a period of sharp volatility in interest rates which interrupted what had been otherwise a relatively stable interest rate regime.

section. Note that ut can be interpreted as the daily return for an agent holding dollars. This follows from solving equation (1) for X_t which gives $X_t = X_{t-1}e^{u_t}$ One dollar buys X_{t-1} crowns at time $t-1$ or $X_{t-1}e^{u_t}$ crowns at time t. Hence ut will be positive if the crown depreciates against the dollar between $t-1$ and t, and vice-versa.

The BDS statistics for the two sub-samples are given in Tables 2 and 3 below.

Table 2. BDS tests on u_t - period 1.

ε	$m=4$	$m=5$	$m=6$	$m=7$	$m=8$
0.110	−0.254	−0.563	−1.008	−1.144	−1.245
0.165	−0.497	−0.878	−0.741	−0.636	−0.303
0.247	−0.305	−0.248	0.045	0.141	0.202

Table 3. BDS tests on u_t - period 2.

ε	$m=4$	$m=5$	$m=6$	$m=7$	$m=8$
0.101	4.542	4.840	4.836	4.719	4.844
0.152	3.041	3.114	3.004	3.165	3.029
0.228	3.087	3.139	3.126	3.046	2.810

Under the null hypothesis of IID, at the 1% significance level, we accept the null if the BDS statistics lie in the interval $(-2.575; 2.575)$. Referring to Tables 1 and 2, it is clear that we can accept the null hypothesis for period 1 and must reject it for period 2: There is strong evidence in favour of weak form market efficiency during period 1 but it is equally clear that this market is not weak form efficient during period 2. The results of this section have shown that the EMH holds and that the data follow a random walk (equation (1) above) only during period 1. In the following section we further explore the period 2 data and develop a suitable model for the exchange rate during this period.

3. A NON-LINEAR TIME SERIES MODEL FOR THE PERIOD 2 EXCHANGE RATE

We know from the preceding section that the exchange rate data are I(1). Consequently, in order to ensure that we are working with stationary data, we will construct a model for the daily 'return' defined by $u_t = ln(X_t/X_{t-1})$ where X_t are the exchange rate data. It is also clear from the BDS statistics in Table 3 that the random walk model suggested by the efficient markets hypothesis does not provide an adequate model for the second period data, because are data are not IID. Considering more general time series models, we next employed standard Box-Jenkins methodology to fit a (linear) mixed auto-regressive moving-average (ARMA) model, such as in equation (3) below, to the period 2 data.[4]

$$u_t = \beta_0 + \sum_{i=1}^{p} \beta_i u_{t-i} + \sum_{i=1}^{q} \gamma_i v_{t-i} + v_t \tag{3}$$

where the v_t are a sequence of zero mean IID random variables and the values for p and q are to be determined. Using BDS statistics to check the residuals of the fitted models for whiteness showed quite clearly that the linear ARMA class of models did not provide a satisfactory explanation for the data. All the BDS statistics were highly significant, indicating unexplained structure in the residuals.

Furthermore, diagnostic tests (Engle, 1982) for auto-regressive conditional heteroscedasticity ($ARCH$) effects proved to be significant and suggest that a $GARCH$ (generalised $ARCH$) may be an appropriate model. It is to this class of models that we now turn our attention. After some preliminary investigation, our preferred model is a $GARCH - M(1,1)$ as given in equation (4) below.

$$\begin{aligned} u_t &= \beta + \delta\sigma_t^2 + v_t \\ v_t \,\Big|\, \Omega_{t-1} &\sim N\left(0, \sigma_t^2\right) \\ \sigma_t^2 &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \sigma_{t-1}^2 \end{aligned} \tag{4}$$

where the ε_t 's are the empirical residuals.

[4] Akaike's information criterion was the criterion adopted for model selection (Akaike, 1978).

Estimation was carried out using the 'Eviews' econometric software. The results are as follows. (Figures in parentheses are t-statistics.)

$$u_t = \underset{(-1.987)}{-17.737\sigma_t^2} + \varepsilon_t$$

$$\sigma_t^2 = \underset{(1.991)}{0.0000205} + \underset{(2.794)}{0.220\varepsilon_{t-1}^2} + \underset{(3.074)}{0.514\sigma_{t-1}^2}$$

Adjusted $R-$squared	-0.009383	Akaikeinfo.criterion	-7.134230
Loglikelihood	600.8164	Durbin $-$ Watson statistic	1.791596

Next, we test whether the *standardised* residuals from this model are IID.[5] Table 4 below gives BDS statistics for the standardised residuals from this model. None of these statistics are significant and there is thus strong evidence to suggest that the standardised residuals are IID. Our model thus seems to capture all the identifiable structure in the data.

Table 4. BDS tests on period 2 standardised $GARCH-M(1,1)$ residuals.

ε	$m=4$	$m=5$	$m=6$	$m=7$	$m=8$
0.106	0.033	-0.243	-0.103	-0.861	-0.785
0.159	0.494	0.253	0.062	0.306	0.291
0.239	0.394	0.361	0.387	0.448	0.360

[5]The residuals, ε_t, will not be IID because of the $GARCH$ model structure. However, the standardised residuals, defined by, will be IID if our model captures all the identifiable structure in the data.

4. Concluding remarks

We have found that, following the floating of the Czech crown, and in a period characterised by relatively stable interest rates coupled with budget surpluses and a strong reserve position, the crown/dollar daily exchange rate exhibited behaviour consistent with that of a (weakly) efficient market for nearly a year. This is an interesting result for a 'new' currency and is in marked contrast to the numerous papers in the literature that reject market efficiency in foreign exchange markets.

Subsequent to the 1997 Asian currency crises, and with a weakening economy and more volatile interest rates, the crown has displayed characteristics of more complicated and non-linearly dependent behaviour, as found in many other exchange rates (Lo and MacKinley, 1988). This was successfully modelled with a $GARCH - M$ formulation.

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