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ON THE STABILITY OF NONLINEAR SYSTEMS WITH RESPECT TO A PART OF THE VARIABLES AND APPLICATION

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Abstract

This paper is concerned with the study of the problem of stability with respect to a part of the variables of nonautonomous systems. A new class of perturbed nonlinear systems is derived to guarantee the global exponential convergence of the solutions toward a small ball centered at the origin. We state some sufficient conditions based on generalized integral inequalities of Gronwall type. Furthermore, as an application we consider the stabilization problem of time-varying perturbed control system.

Keywords: Gronwall inequalities, nonautonomous systems, partial stability, practical partial stability.

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