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## ON THE STABILITY OF NONLINEAR SYSTEMS WITH RESPECT TO A PART OF THE VARIABLES AND APPLICATION

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## Abstract

This paper is concerned with the study of the problem of stability with respect to a part of the variables of nonautonomous systems. A new class of perturbed nonlinear systems is derived to guarantee the global exponential convergence of the solutions toward a small ball centered at the origin. We state some sufficient conditions based on generalized integral inequalities of Gronwall type. Furthermore, as an application we consider the stabilization problem of time-varying perturbed control system.

**Keywords:** Gronwall inequalities, nonautonomous systems, partial stability, practical partial stability.

2020 Mathematics Subject Classification: 34D20, 37B25, 37B55.

## References

- D. Bainov and P. Simenov, Integral Inequalities and Applications (Dordrecht, Kluwer Academic Publishers, 1992).
- R. Bellman, The stability of solutions of linear differential equations, Duke Math. J. 10 (1943) 643-647. https://doi.org/10.1215/s0012-7094-43-01059-2
- [3] M. Benjemaa, W. Gouadri and M.A. Hammami, New results on the uniform exponential stability of nonautonomous perturbed dynamical systems, Int. J. Robust and Nonlinear Control 12 (2021) 5563–5579. https://doi.org/10.1002/rnc.5550
- [4] A. Ben Makhlouf and M.A. Hammami, Nonlinear inequality and application to global asymptotic stability of perturbed systems, Math. Models Meth. Appl. Sci. 38 (2015) 2496-2505. https://doi.org/10.1002/mma.3236

- [5] A. Ben Abdallah, M. Dlala and M.A. Hammami, Exponential stability of perturbed nonlinear systems, Nonlinear Dynamics and Syst. Theory 5 (2005) 357–367.
- [6] B. Ben Hamed, I. Ellouze and M.A. Hammami, Practical uniform stability of nonlinear differential delay equations, Mediterranean J. Math. 8 (2011) 603–616. https://doi.org/10.1007/s00009-010-0083-7
- B. Benaser, K. Boukerrioua, M. Defoort, M. Djemai, M.A. Hammami, and T.M.L. Kirati, Sufficient conditions for uniform exponential stability and h-stability of some classes of dynamic equations on arbitrary time scales, Nonlinear Anal. Hybrid Syst. 32 (2019) 54-64.

https://doi.org/10.1016/j.nahs.2018.10.009

- [8] R.T. Bupp, D.S. Bernstein and A. Benchmark, Problem for nonlinear and robust control, Int. J. Robust and Nonlinear Control 8 (1992) 489–502. https://doi.org/10.1002/(sici)1099-1239(19980415/30)8:4/5j307::aidrnc354¿3.0.co;2-7
- [9] T. Caraballo, F. Ezzine, M. Hammami and L. Mchiri, Practical stability with respect to a part of variables of stochastic differential equations, Stochastics Int. J. Prob. and Stochastic Process. 6 (2020) 1–18. https://doi.org/10.1080/17442508.2020.1773826
- [10] T. Caraballo, F. Ezzine and M.A. Hammami, On the exponential stability of stochastic perturbed singular systems in mean square, Appl. Math. Optim. 84 (2021) 1–23. https://doi.org/10.1007/s00245-020-09734-8
- T. Caraballo, F. Ezzine and M.A. Hammami, New stability criteria for stochastic perturbed singular systems in mean square, Nonlinear Dynamics 105 (2021) 241– 256. https://doi.org/10.1007/s11071-021-06620-y
- [12] T. Caraballo, F. Ezzine and M.A. Hammami, Partial stability analysis of stochastic differential equations with a general decay rate, J. Eng. Math. 1 (2021) 1–17. https://doi.org/10.1007/s10665-021-10164-w
- T. Caraballo, F. Ezzine and M.A. Hammami, Practical stability with respect to a part of the variables of stochastic differential equations driven by G-Brownian motion, J. Dynamical and Control Systems 33 (2021) 1–19. https://doi.org/10.1007/s10883-022-09593-2
- T. Caraballo, F. Ezzine and M.A. Hammami, Stability with respect to a part of the variables of stochastic nonlinear systems driven by G-Brownian motion, Int. J. Control 33 (2022) 1–14. https://doi.org/10.1080/00207179.2022.2070548
- [15] S.S. Dragomir, Some Gronwall Type Inequalities and Applications (Nova Science Publishers, 2003).
- [16] S.S. Dragomir and S. Sever, Some Hermite-Hadamard type integral inequalities for convex functions defined on convex bodies, J. Appl. Anal. 26 (2020) 67–77. https://doi.org/10.1515/jaa-2020-2005

- [17] F. Ezzine and M. Hammami, Growth conditions for the stability of a class of timevarying perturbed singular systems, Kybernetika 58 (2022) 1–24. https://doi.org/10.14736/kyb-2022-1-0001
- I. Ellouze and M.A. Hammami, A separation principle of time-varying dynamical systems: A practical stability approach, Math. Modelling and Anal. 12 (2007) 297– 308. https://doi.org/10.3846/1392-6292.2007.12.297-308
- [19] I. Ellouze, H. Damak and M.A. Hammami, A separation principle of time-varying nonlinear dynamical systems, J. Diff. Equ. Control Process. 12 (2013) 36–49.
- [20] T.H. Gronwall, Note on the derivatives with respect to a parameter of the solutions of a system of differential equations, Ann. Math. 20 (1919) 293–296. https://doi.org/10.2307/1967124
- [21] B. Ghanmi, N. Hadj Taieb and M.A. Hammami, Growth conditions for exponential stability of time-varying perturbed systems, Int. J. Control 86 (2013) 1086–1097. https://doi.org/10.1080/00207179.2013.774464
- [22] M. Hammi and M.A. Hammami, Non-linear integral inequalities and applications to asymptotic stability, IMA J. Math. Control and Inform. 32 (2015) 717–735. https://doi.org/10.1093/imamci/dnu016
- [23] N. Hadj Taieb and M.A. Hammami, Some new results on the global uniform asymptotic stability of time-varying dynamical systems, IMA J. Math. Control and Inform. 32 (2017) 1–22. https://doi.org/10.1093/imamci/dnx006
- [24] W.M. Haddad and V. Chellaboina, Nonlinear Dynamical Systems and Control (Princeton University Press, 2007).
- [25] I. Karafyllis and Z.P. Jiang, Stability and Stabilization of Nonlinear Systems (Springer Verlag, London, 2011).
- [26] A. Kashuri and R. Liko, Generalization of different type integral inequalities for generalized preinvex Godunova-Levin functions, J. Appl. Anal. 24 (2018) 211–221. https://doi.org/10.1515/jaa-2018-0020
- [27] H.K. Khalil, Nonlinear Systems, 2nd edition, (Mac-Millan, 1996).
- [28] K.Y. Lum, D.S. Bernstein and V.T. Coppola, Global stabilization of the spinning top with mass imbalance, Dynamics and Stability of Systems 10 (1995) 339–365. https://doi.org/10.1080/02681119508806211
- [29] N. Rouche, P. Habets and M. Lalog, Stability Theory by Liapunov's Direct Method (New York, Springer, 1977).
- [30] V.A. Sinitsyn, On stability of solution in inertial navigation problem, Certain Problems on Dynamics of Mechanical Systems (1991) 46–50.
- [31] V.I. Vorotnikov, Partial Stability and Control (Birkhauser Boston, 1998).

- [32] R. Vrabel, Local null controllability of the control-affine nonlinear systems with timevarying disturbances. Direct calculation of the null controllable region, Europ. J. Control 40 (2018) 80–86. https://doi.org/10.1016/j.ejcon.2017.12.004
- [33] V.I. Zubov, The Dynamics of Controlled Systems (Moscow, Vysshaya Shkola, 1982).

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