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PENALTIES, LAGRANGE MULTIPLIERS AND NITSCHE MORTARING

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Abstract

Penalty methods, augmented Lagrangian methods and Nitsche mortaring are well known numerical methods among the specialists in the related areas optimization and finite elements, respectively, but common aspects are rarely available. The aim of the present paper is to describe these methods from a unifying optimization perspective and to highlight some common features of them.

Keywords: augmented Lagrangian, penalty method, domain decomposition, Nitsche mortaring, finite elements.

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