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MULTIVALUED ANISOTROPIC PROBLEM WITH NEUMANN BOUNDARY CONDITION INVOLVING DIFFUSE RADON MEASURE DATA AND VARIABLE EXPONENT

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Abstract

We study a nonlinear anisotropic elliptic problem with homogeneous Neumann boundary condition governed by a general anisotropic operator with variable exponents and diffuse Radon measure data that is the Radon measure which does not charge the sets of zero $p(\cdot)$-capacity. We firstly prove the existence of renormalized solutions. Secondly, we show an equivalence between renormalized solution and entropy solution. Thirdly, we end by proving an uniqueness result of entropy solution.

Keywords: Neumann boundary, anisotropic Sobolev spaces, renormalized solution, entropy solution, maximal monotone graph, Radon diffuse measure, Marcinkiewicz spaces, $p(\cdot)$-capacity.

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