

**RANDOM INTEGRAL GUIDING FUNCTIONS WITH
APPLICATION TO RANDOM DIFFERENTIAL
COMPLEMENTARITY SYSTEMS**

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Abstract

By applying the random topological degree we develop the methods of random smooth and nonsmooth integral guiding functions and use them for the study of random differential inclusions in finite dimensional spaces.

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Some existence theorems of random periodic solutions are presented. It is shown how the abstract results can be applied to study the random differential complementarity systems arising, in particular, from random survival models.

Keywords: random differential inclusion, random periodic solution, random differential complementarity system, random guiding function, random topological degree.

2010 Mathematics Subject Classification: 34F05, 34A38, 34A60, 34C25, 47H11.

REFERENCES

- [1] S. Adly and D. Goeleven, *A stability theory for second-order nonsmooth dynamical systems with application to friction problems*, J. Math. Pures Appl. **83** (2004) 17–51. doi:10.1016/S0021-7824(03)00071-0
- [2] J. Andres and L. Górniewicz, *Random topological degree and random differential inclusions*, Topol. Meth. Nonl. Anal. **40** (2012) 337–358.
- [3] A.V. Arutyunov and V. Obukhovskii, *Convex and Set-Valued Analysis, Selected Topics*, De Gruyter Graduate (Walter de Gruyter, Berlin-Boston, 2016). doi:10.1515/9783110460308
- [4] V. Barbu, *Nonlinear Semigroups and Differential Equations in Banach Spaces* (Noordhoff International Publishing, Leyden, 1976).
- [5] Yu.G. Borisovich, B.D. Gelman, A.D. Myshkis and V.V. Obukhovskii, *Introduction to the Theory of Multivalued Maps and Differential Inclusions*, (Russian) Second edition (Librokom, Moscow, 2011).
- [6] M.K. Camlibel, *Complementarity Methods in the Analysis of Piecewise Linear Dynamical Systems*, Ph.D. thesis, Center for Economic Research (Tilburg University, The Netherlands, 2001).
- [7] M.K. Camlibel, J.-S. Pang and J. Shen, *Lyapunov stability of complementarity and extended systems*, SIAM J. Optim. **17** (2006) 1056–1101. doi:10.1137/050629185
- [8] A. Capietto and F. Zanolin, *A continuation theorem for the periodic bvp in flow-invariant ENRs with applications*, J. Diff. Equ. **83** (1990) 244–276. doi:10.1016/0022-0396(90)90058-W
- [9] C. Castaing and M. Valadier, *Convex Analysis and Measurable Multifunctions*, Lecture Notes in Mathematics, 580 (Springer-Verlag, Berlin-New York, 1977). doi:10.1007/BFb0087685
- [10] F.H. Clarke, *Optimization and Nonsmooth Analysis*, Second edition, Classics in Applied Mathematics 5 (Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1990). doi:10.1137/1.9781611971309

- [11] R.W. Cottle and J.C. Yao, *Pseudo-monotone complementarity problems in Hilbert space*, J. Opt. Theory Appl. **75** (1992) 281–295.
doi:10.1007/BF00941468
- [12] F.S. De Blasi, L. Górniewicz and G. Pianigiani, *Topological degree and periodic solutions of differential inclusions*, Nonlinear Anal. **37** (1999), Ser. A, Theory Methods, 217–243.
doi:10.1016/S0362-546X(98)00044-3
- [13] K. Deimling, *Multivalued Differential Equations*, de Gruyter Series in Nonlinear Analysis and Applications 1 (Walter de Gruyter, Berlin-New York, 1992).
- [14] V.F. Dem'yanov and L.V. Vasil'ev, *Nondifferentiable Optimization*. Translation Series in Mathematics and Engineering, Optimization Software, Inc., Publications Division (New York, 1985).
- [15] A. Fonda, *Guiding functions and periodic solutions to functional differential equations*, Proc. Amer. Math. Soc. **99** (1987) 79–85.
- [16] D. Gabor and W. Kryszewski, *A global bifurcation index for set-valued perturbations of Fredholm operators*, Nonlinear Anal. TMA **73** (2010) 2714–2736.
doi:10.1016/j.na.2010.06.055
- [17] R.E. Gaines and J.L. Mawhin, *Coincidence Degree and Nonlinear Differential Equations*, Lecture Notes in Mathematics, 568 (Springer-Verlag, Berlin-New York, 1977).
doi:10.1007/BFb0089537
- [18] L. Górniewicz, *Topological Fixed Point Theory of Multivalued Mappings*, 2nd edition, Topological Fixed Point Theory and Its Applications, 4 (Springer, Dordrecht, 2006).
- [19] L. Górniewicz and S. Plaskacz, *Periodic solutions of differential inclusions in \mathbb{R}^n* , Boll. UMI **7-A** (1993) 409–420.
- [20] W.P.H. Heemels, *Linear Complementarity Systems: A Study in Hybrid Dynamics*, Ph.D. thesis, Department of Electrical Engineering (Eindhoven University of Technology, 1999).
- [21] C. Henry, *An existence theorem for a class of differential equations with multivalued right-hand side*, J. Math. Anal. Appl. **42** (1973) 179–186.
doi:10.1016/0022-247X(73)90192-3
- [22] D. Hipfel, *The nonlinear differential complementarity problem*, PhD Thesis (Department of Mathematical Sciences, Rensselaer Polytechnic Institute, 1993).
- [23] S. Hu and N.S. Papageorgiou, *Handbook in Multivalued Analysis, Vol. I: Theory*, Kluwer AP (Dordrecht-Boston-London, 1997).
- [24] M. Kamenskii, V. Obukhovskii and P. Zecca, *Condensing Multivalued Maps and Semilinear Differential Inclusions in Banach Spaces*, de Gruyter Series in Nonlinear Analysis and Applications 7 (Walter de Gruyter, Berlin-New York, 2001).
- [25] M. Kisielewicz, *Differential Inclusions and Optimal Control*, Mathematics and its Applications (East European Series), 44. Kluwer Academic Publishers Group, Dordrecht; PWN—Polish Scientific Publishers, Warsaw, 1991.

- [26] S. Kornev and V. Obukhovskii, *On some developments of the method of integral guiding functions*, *Funct. Differ. Equ.* **12** (2005) 303–310.
- [27] S. Kornev, V. Obukhovskii and J.-C. Yao *On asymptotics of solutions for a class of functional differential inclusions*, *Discus. Math. Diff. Incl. Control and Optimiz.* **34** (2014) 219–227.
doi:10.7151/dmdico.1165
- [28] S. Kornev, V. Obukhovskii and P. Zecca, *Guiding functions and periodic solutions for inclusions with causal multioperators*, *Appl. Anal.* **96** (2017) 418–428.
doi:10.1080/00036811.2016.1139088
- [29] M.A. Krasnosel'skii and A.I. Perov, *On a certain principle of existence of bounded, periodic and almost periodic solutions of systems of ordinary differential equations*, *Dokl. Akad. Nauk SSSR* **123** (1958) 235–238 (in Russian).
- [30] M.A. Krasnosel'skii, *The Operator of Translation Along the Trajectories of Differential Equations*. *Translations of Mathematical Monographs* 19 (Amer. Math. Soc., Providence, R.I., 1968).
- [31] W. Kryszewski, *Properties of Set-Valued Mappings*, Univ. N. Copernicus Publishing (Toruń, 1997).
- [32] X. Liu, *Analysis of some dynamical systems inspired by ecological interactions*, PhD dissertation (Univ. Helsinki, 2017).
- [33] N.V. Loi, *Method of guiding functions for differential inclusions in a Hilbert space*, *Diff. Equ.* **46** (2010) 1438–1447.
doi:10.1134/S0012266110100071
- [34] N.V. Loi, M.Q. Vu and Ph.T. Cuong, *Nonlocal problem for differential complementarity systems*, *Appl. Math. Comp.* **265** (2015) 799–806.
doi:10.1016/j.amc.2015.05.143
- [35] J. Mawhin, *Periodic solutions of nonlinear functional differential equations*, *J. Diff. Equ.* **10** (1971) 240–261.
doi:10.1016/0022-0396(71)90049-0
- [36] V. Obukhovskii, P. Zecca, N.V. Loi and S. Kornev, *Method of Guiding Functions in Problems of Nonlinear Analysis*, *Lecture Notes in Math*, 2076 (Springer-Verlag, Berlin-Heidelberg, 2013).
- [37] G. Oshanin, O. Vasilyev, P.L. Krapivsky and J. Klafter, *Survival of an evasive prey*, *Proc. Natl. Acad. Sci. USA* **106** (2009) 13696–13701.
doi:10.1073/pnas.0904354106
- [38] J.-S. Pang and D.E. Steward, *Differential variational inequalities*, *Math. Program., Ser. A* **113** (2008) 345–424.

Received 12 June 2018

Accepted 4 July 2018