

## ON PROPERTIES OF SET-VALUED INTEGRALS DRIVEN BY MARTINGALES AND SET-VALUED STOCHASTIC EQUATIONS

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### Abstract

In the paper we study properties of stochastic integrals of Aumann type driven by quadratic variation process and set-valued Itô integral with respect to martingale. Next, the existence, uniqueness and convergence properties of solutions to set-valued stochastic differential equations with respect to such integrators are investigated. The results obtained in the paper generalize conclusions dealing with this topic known both in deterministic and stochastic cases.

**Keywords:** set-valued function, set-valued stochastic integral, set-valued stochastic differential equation.

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