

**MULTIPLE SOLUTIONS FOR DIRICHLET IMPULSIVE
FRACTIONAL DIFFERENTIAL INCLUSIONS INVOLVING
THE p -LAPLACIAN WITH TWO PARAMETERS**

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Abstract

In this paper, the authors establish the existence of at least three weak solutions for impulsive differential inclusions involving two parameters and the p -Laplacian and having Dirichlet boundary conditions. Their approach is based variational methods and critical point theory.

Keywords: impulsive fractional differential inclusions, p -Laplacian, variational methods.

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