

## UNIFIED THEORY OF TESTING HYPOTHESES IN MIXED LINEAR MODELS

ANDRZEJ MICHALSKI

*Department of Mathematics*  
*Wrocław University of Environmental and Life Sciences*  
*50-357 Wrocław, Poland*

**e-mail:** apm.mich@gmail.com

*Dedicated to Professor Roman Zmyślony on the occasion of this 70th Birthday*

### Abstract

In the paper an effective method of testing hypotheses for fixed effects and variance components in mixed linear normal models is presented. A new idea in the problem of testing hypotheses for parameters in mixed linear models was born in the construction of test for vanishing of single variance component (Michalski and Zmyślony, 1996) [9]. This test has been based on the decomposition of quadratic form of the locally best quadratic unbiased estimator of this components. The F-ratio test rejects null hypothesis if the ratio of positive and negative part of the corresponding estimator is sufficiently large. Although the construction of this exact test requires quite strong assumptions (covariance matrices commute after using the usual invariance procedure with respect to the group of translations), for many classic models, analysis of variance and regression models, we can successfully apply this idea to get the classical tests for testing hypotheses about the parameters of the corresponding model, often with larger values of the power function. In the case of testing the fixed effects is sufficient to replace linear hypothesis by the equivalent square hypothesis. Next, we consider the respective functions of the model parameters, which are squarely estimable connected with the null hypothesis. This article is a review of the most important results obtained by Michalski and Zmyślony (1996 [9], 1999 [10]) and by Gaşiorek *et al.* (2000) [2] using this idea.

**Keywords:** mixed linear models, variance components, quadratic estimation, testing hypotheses, block designs, correlations.

**2010 Mathematics Subject Classification:** 62F03, 62K05, 62J10.

## REFERENCES

- [1] M. Fonseca, J.T. Mexia and R. Zmyślony, *Estimation and tests for variance components in cross nested orthogonal models*, Discuss. Math. Probab. and Stat. **23** (2003) 173–201.
- [2] E. Gąsiorek, A. Michalski and R. Zmyślony, *Tests of independence of normal random variables with known and unknown variance ratio*, Discuss. Math. Probab. and Stat. **20** (2000) 233–247.
- [3] S. Gnot and J. Kleffe, *Quadratic estimation in mixed linear models with two variance components*, J. Statist. Plann. Inference **8** (1983) 267–279.
- [4] S. Gnot and A. Michalski, *Tests based on admissible estimators in two variance components models*, Statistics **25** (1994) 213–223.
- [5] E.L. Lehmann, *Testing statistical hypotheses* (Wiley, New York, 1986).
- [6] T. Mathew, *Optimum invariant tests in mixed linear models with two variance components*, in: *Statistical Data Analysis and Inference*, Y. Dodge (Ed(s)), (North-Holland, 1989) 381–388.
- [7] A. Michalski, *On some aspects of the optimal statistical inference of variance components in mixed linear models*, Tatra Mountains Mathematical Publications **26** (2003) 1–21.
- [8] A. Michalski, *Bayesian and generalized confidence intervals on variance ratio and on the variance component in mixed linear models*, Discuss. Math. Probab. and Stat. **29** (2009) 5–29.
- [9] A. Michalski and R. Zmyślony, *Testing hypotheses for variance components in mixed linear models*, Statistics **27** (1996) 297–310.
- [10] A. Michalski and R. Zmyślony, *Testing hypotheses for linear functions of parameters in mixed linear models*, Tatra Mountains Mathematical Publications **17** (1999) 103–110.
- [11] A. Olsen, J. Seely and D. Birkes, *Invariant quadratic unbiased estimation for two variance components*, Ann. Statist **4** (1976) 878–890.
- [12] C.A. Rao, *Linear statistical inference and its applications* (2nd ed. Wiley, New York, 1973).
- [13] J. Seely, *Quadratic subspaces and completeness*, Ann. Math. Statist **42** (1971) 710–721.
- [14] R. Zmyślony, *On estimation parameters in linear models*, Zastosowania Matematyki **15** (1976) 271–276.
- [15] R. Zmyślony, *Completeness for a family of normal distributions*, Mathematical Statistics, Banach Center Publications, Warsaw, PWN - Polish Scientific Publishers **6** (1980) 355–357.

Received 25 August 2017  
Accepted 20 September 2017