

## ON NONNEGATIVE MINIMUM BIASED ESTIMATION IN THE LINEAR REGRESSION MODELS

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### Abstract

The problem of the nonnegative estimation of the parametric function  $\gamma(\beta, \sigma) = \beta' H \beta + h \sigma^2$  in the linear regression model  $\mathcal{M}\{y, X\beta, \sigma^2 I\}$ , where  $H$  is a nonnegative definite matrix and  $h$  is a nonnegative scalar, attracted attention of many researchers. S. Gnot, G. Trenkler and R. Zmyślony [J. Multivar. Anal. 54 (1995), 113–125] proposed an approach in which  $\gamma$  is estimated by a quadratic form  $y' A y$ , where  $A$  is a nonnegative definite matrix that satisfies an appropriate optimality criterion associated with minimizing the bias of the estimator.

In the paper, we revisit this approach to estimating  $\gamma$ . In particular, we discuss various methods of computing the matrix  $A$ , which in the general case is not given explicitly.

**Keywords:** linear regression model, nonnegative minimum biased estimators, mean squared error.

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