

ZERO-DIVISOR GRAPHS OF REDUCED RICKART *-RINGS

A.A. PATIL

Department of Mathematics
Garware College of Commerce, Pune-411004, India

e-mail: avipmj@gmail.com

AND

B.N. WAPHARE

Center for Advanced Studies in Mathematics
Department of Mathematics
Savitribai Phule Pune University, Pune-411007, India

e-mail: waphare@yahoo.com
bnwaph@math.unipune.ac.in

Abstract

For a ring A with an involution $*$, the *zero-divisor graph* of A , $\Gamma^*(A)$, is the graph whose vertices are the nonzero left zero-divisors in A such that distinct vertices x and y are adjacent if and only if $xy^* = 0$. In this paper, we study the zero-divisor graph of a Rickart $*$ -ring having no nonzero nilpotent element. The distance, diameter, and cycles of $\Gamma^*(A)$ are characterized in terms of the collection of prime strict ideals of A . In fact, we prove that the clique number of $\Gamma^*(A)$ coincides with the cellularity of the hull-kernel topological space $\Sigma(A)$ of the set of prime strict ideals of A , where cellularity of the topological space is the smallest cardinal number m such that every family of pairwise disjoint non-empty open subsets of the space have cardinality at most m .

Keywords: reduced ring, Rickart $*$ -ring, zero-divisor graph, prime strict ideals.

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