

**SOME HOMOLOGICAL PROPERTIES
OF SKEW *PBW* EXTENSIONS ARISING IN
NON-COMMUTATIVE ALGEBRAIC GEOMETRY**

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Abstract

In this short paper we study for the skew *PBW* (Poincar-Birkhoff-Witt) extensions some homological properties arising in non-commutative algebraic geometry, namely, Auslander-Gorenstein regularity, Cohen-Macaulayness and strongly noetherianity. Skew *PBW* extensions include a considerable number of non-commutative rings of polynomial type such that classical *PBW* extensions, quantum polynomial rings, multiplicative analogue of the Weyl algebra, some Sklyanin algebras, operator algebras, diffusion algebras, quadratic algebras in 3 variables, among many others. Parametrization of the point modules of some examples is also presented.

Keywords: Auslander regularity condition, Cohen-Macaulay rings, strongly noetherian algebras, skew *PBW* extensions, filtered-graded rings, point modules.

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