

GENERALIZED LEHMANN FAMILY FOR META ANALYSIS  
BASED UPON SUMMARY RECEIVER OPERATING  
CHARACTERISTIC CURVES

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**Abstract**

In this paper, we consider the modelling of Summary Receiver Operating Characteristic (SROC) curve used in meta-analysis of diagnostic studies. This is done through Generalized Lehmann model which relates the log-sensitivity and the log false positive rate across various studies. The nuisance parameters (study specific false positive rates) are eliminated through the use of profile likelihood. The estimation for the parameters of the Generalized Lehmann family has been carried out using adjusted profile likelihood. This model is extended further to accommodate unobserved heterogeneity by allowing the constant of proportionality to vary across studies.

**Keywords:** meta analysis, sensitivity, false positive rate, generalized Lehmann family, profile likelihood, adjusted profile likelihood, SROC.

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