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ON NON-EXISTENCE OF MOMENT ESTIMATORS OF THE GED POWER PARAMETER

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Abstract

We reconsider the problem of the power (also called shape) parameter estimation within symmetric, zero-mean, unit-variance one-parameter Generalized Error Distribution family. Focusing on moment estimators for the parameter in question, through extensive Monte Carlo simulations we analyze the probability of non-existence of moment estimators for small and moderate samples, depending on the shape parameter value and the sample size. We consider a nonparametric bootstrap approach and prove its consistency. However, despite its established asymptotics, bootstrap does not substantially improve the statistical inference based on moment estimators once they fall into the *non-existence area* in case of small and moderate sample sizes.

Keywords: Generalized Error Distribution, nonparametric bootstrap, bootstrap consistency, moment estimator, power parameter.

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