

THE ASYMPTOTIC TRACE NORM OF RANDOM CIRCULANTS AND THE GRAPH ENERGY

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Abstract

We compute the expected normalized trace norm (matrix/graph energy) of random symmetric band circulant matrices and graphs in the limit of large sizes, and obtain explicit bounds on the rate of convergence to the limit, and on the probabilities of large deviations. We also show that random symmetric band Toeplitz matrices have the same limit norm assuming that their band widths remain small relative to their sizes. We compare the limit norms across a range of related random matrix and graph ensembles.

Keywords: random matrix, graph energy, matrix energy, circulant, Toeplitz matrix, band matrix, Dirichlet kernel, non-uniform Berry-Esseen estimate, Talagrand concentration inequality.

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