

WEAKLY PRECOMPACT OPERATORS ON $C_b(X, E)$ WITH THE STRICT TOPOLOGY

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Abstract

Let X be a completely regular Hausdorff space, E and F be Banach spaces. Let $C_b(X, E)$ be the space of all E -valued bounded continuous functions on X , equipped with the strict topology β . We study weakly precompact operators $T : C_b(X, E) \rightarrow F$. In particular, we show that if X is a paracompact k -space and E contains no isomorphic copy of l^1 , then every strongly bounded operator $T : C_b(X, E) \rightarrow F$ is weakly precompact.

Keywords: spaces of vector-valued continuous functions, strict topologies, operator measures, strongly bounded operators, weakly precompact operators.

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