Discussiones Mathematicae
Differential Inclusions, Control and Optimization 36 (2016) 79–80
doi:10.7151/dmdico.1181

TOPOLOGICAL PROPERTIES OF SOME SPACES OF CONTINUOUS OPERATORS

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Abstract

Let X be a completely regular Hausdorff space, E and F be Banach spaces. Let $C_b(X,E)$ be the space of all E-valued bounded continuous functions on X, equipped with the strict topology β. We study topological properties of the space $\mathcal{L}_\beta(C_b(X,E),F)$ of all $(\beta,\|\cdot\|_F)$-continuous linear operators from $C_b(X,E)$ to F, equipped with the topology τ_s of simple convergence. If X is a locally compact paracompact space (resp. a P-space), we characterize τ_s-compact subsets of $\mathcal{L}_\beta(C_b(X,E),F)$ in terms of properties of the corresponding sets of the representing operator-valued Borel measures. It is shown that the space $(\mathcal{L}_\beta(C_b(X,E),F),\tau_s)$ is sequentially complete if X is a locally compact paracompact space.

Keywords: spaces of vector-valued continuous functions, strict topologies, operator measures, topology of simple convergence, continuous operators.

2010 Mathematics Subject Classification: 46G10, 46E10, 46A70.

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Received 19 February 2016
Revised 17 March 2016