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EFFECTIVE ENERGY INTEGRAL FUNCTIONALS FOR THIN FILMS WITH THREE DIMENSIONAL BENDING MOMENT IN THE ORLICZ-SOBOLEV SPACE SETTING

Włodzimierz Laskowski

School of Mathematics
West Pomeranian University of Technology
Al. Piastów 48, 70–311 Szczecin, Poland

e-mail: wlaskowski@zut.edu.pl

AND

Hong Thai Nguyen

Institute of Mathematics
Szczecin University
ul. Wielkopolska 15, 70–451 Szczecin, Poland

e-mail: nguyenhthaimathuspl@yahoo.com

Abstract

In this paper we consider an elastic thin film $\omega \subset \mathbb{R}^2$ with the bending moment depending also on the third thickness variable. The effective energy functional defined on the Orlicz-Sobolev space over ω is described by Γ-convergence and $3D$-$2D$ dimension reduction techniques. Then we prove the existence of minimizers of the film energy functional. These results are proved in the case when the energy density function has the growth prescribed by an Orlicz convex function M. Here M is assumed to be non-power-growth-type and to satisfy the conditions Δ_2 and ∇_2.

Keywords: Γ-convergence, 3D-2D dimension reduction, quasiconvex relaxation, minimizers of variational integral functionals, thin films, elastic membranes, effective energy integral functional, bulk and surface energy, equilibrium states of the film, non-power-growth-type bulk energy density, reflexive Orlicz and Orlicz-Sobolev spaces.

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