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ON THE MUTUALLY NON ISOMORPHIC $\ell_p(\ell_q)$ SPACES, A SURVEY

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Abstract

In this note we survey the partial results needed to show the following general theorem: $\{\ell_p(\ell_q) : 1 \leq p, q \leq +\infty\}$ *is a family of mutually non isomorphic Banach spaces.* We also comment some related facts and open problems.

Keywords: Banach spaces, isomorphic spaces, complemented subspace.

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