

**PROPERTIES OF THE GENERALIZED NONLINEAR LEAST
SQUARES METHOD APPLIED FOR FITTING
DISTRIBUTION TO DATA**

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Abstract

We introduce and analyze a class of estimators for distribution parameters based on the relationship between the distribution function and the empirical distribution function. This class includes the nonlinear least squares estimator and the weighted nonlinear least squares estimator which has been used in parameter estimation for lifetime data (see e.g. [6, 8]) as well as the generalized nonlinear least squares estimator proposed in [3]. Sufficient conditions for consistency and asymptotic normality are given. Capability and limitations are illustrated by simulations.

Keywords: generalized least squares, distribution fitting, generalized method of moments.

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