

## HYBRID FRACTIONAL INTEGRO-DIFFERENTIAL INCLUSIONS

SOTIRIS K. NTOUYAS <sup>a,b</sup>, SORASAK LAOPRASITTICHOK <sup>c</sup>

AND

JESSADA TARIBOON <sup>c,1</sup>

<sup>a</sup>*Department of Mathematics, University of Ioannina*  
*451 10 Ioannina, Greece*

<sup>b</sup>*Nonlinear Analysis and Applied Mathematics (NAAM)-Research Group*  
*Department of Mathematics, Faculty of Science, King Abdulaziz University*  
*P.O. Box 80203, Jeddah 21589, Saudi Arabia*

<sup>c</sup>*Nonlinear Dynamic Analysis Research Center*  
*Department of Mathematics, Faculty of Applied Science*  
*King Mongkut's University of Technology North Bangkok*  
*Bangkok 10800, Thailand*

**e-mail:** [sntouyas@uoi.gr](mailto:sntouyas@uoi.gr)  
[sorasak\\_kmutnb@hotmail.com](mailto:sorasak_kmutnb@hotmail.com)  
[jessada.t@sci.kmutnb.ac.th](mailto:jessada.t@sci.kmutnb.ac.th)

### Abstract

In this paper we study an existence result for initial value problems for hybrid fractional integro-differential inclusions. A hybrid fixed point theorem for a sum of three operators due to Dhage is used. An example illustrating the obtained result is also presented.

**Keywords:** fractional differential equations, hybrid differential inclusions, fixed point theorems.

**2010 Mathematics Subject Classification:** 34A60, 34A08, 34A12.

### REFERENCES

- [1] A.A. Kilbas, H.M. Srivastava and J.J. Trujillo, *Theory and Applications of Fractional Differential Equations* (North-Holland Mathematics Studies, 204, Elsevier Science B.V., Amsterdam, 2006).

---

<sup>1</sup>Corresponding author.

- [2] K.S. Miller and B. Ross, *An Introduction to the Fractional Calculus and Fractional Differential Equations* (Wiley and Sons, New York, 1993).
- [3] V. Lakshmikantham, S. Leela and J. Vasundhara Devi, *Theory of Fractional Dynamic Systems* (Cambridge Academic Publishers, Cambridge, 2009).
- [4] V. Lakshmikantham and A.S. Vatsala, *Basic theory of fractional differential equations*, *Nonlinear Anal.* **69** (8) (2008), 2677–2682. doi:10.1016/j.na.2007.08.042
- [5] I. Podlubny, *Fractional Differential Equations* (Academic Press, San Diego, 1999).
- [6] J. Sabatier, O.P. Agrawal and J.A.T. Machado (Eds.), *Advances in Fractional Calculus: Theoretical Developments and Applications in Physics and Engineering* (Springer, Dordrecht, 2007). doi:10.1007/978-1-4020-6042-7
- [7] B. Ahmad, *Existence of solutions for irregular boundary value problems of nonlinear fractional differential equations*, *Appl. Math. Lett.* **23** (2010), 390–394. doi:10.1016/j.aml.2009.11.004
- [8] B. Ahmad and J.J. Nieto, *Existence results for a coupled system of nonlinear fractional differential equations with three-point boundary conditions*, *Comput. Math. Appl.* **58** (2009), 1838–1843. doi:10.1016/j.camwa.2009.07.091
- [9] P. Thiramanus, S.K. Ntouyas and J. Tariboon, *Existence and uniqueness results for Hadamard-type fractional differential equations with nonlocal fractional integral boundary conditions*, *Abstr. Appl. Anal.* Volume 2014, Article ID 902054, 9 pages.
- [10] J. Tariboon, S.K. Ntouyas and W. Sudsutad, *Fractional integral problems for fractional differential equations via Caputo derivative*, *Adv. Differ. Equ.* **2014** (2014), 181. doi:10.1186/1687-1847-2014-181
- [11] B. Ahmad, S.K. Ntouyas and A. Alsaedi, *New existence results for nonlinear fractional differential equations with three-point integral boundary conditions*, *Adv. Differ. Equ.* (2011), Art. ID 107384, pp. 11.
- [12] B. Ahmad and S.K. Ntouyas, *A four-point nonlocal integral boundary value problem for fractional differential equations of arbitrary order*, *Electron. J. Qual. Theory Differ. Equ.* (2011) No. 22, pp. 15. doi:10.14232/ejqtde.2011.1.22
- [13] B. Ahmad and S. Sivasundaram, *Existence and uniqueness results for nonlinear boundary value problems of fractional differential equations with separated boundary conditions*, *Commun. Appl. Anal.* **13** (2009), 121–228.
- [14] B. Ahmad and S. Sivasundaram, *On four-point nonlocal boundary value problems of nonlinear integro-differential equations of fractional order*, *Appl. Math. Comput.* **217** (2010), 480–487. doi:10.1016/j.amc.2010.05.080
- [15] B. Ahmad, S.K. Ntouyas and A. Alsaedi, *Existence theorems for nonlocal multi-valued Hadamard fractional integro-differential boundary value problems*, *J. Ineq. Appl.* **2014** (2014), 454. doi:10.1186/1029-242X-2014-454
- [16] Y. Zhao, S. Sun, Z. Han and Q. Li, *Theory of fractional hybrid differential equations*, *Comput. Math. Appl.* **62** (2011), 1312–1324. doi:10.1016/j.camwa.2011.03.041

- [17] S. Sun, Y. Zhao, Z. Han and Y. Li, *The existence of solutions for boundary value problem of fractional hybrid differential equations*, Commun. Nonlinear Sci. Numer. Simul. **17** (2012), 4961–4967. doi:10.1016/j.cnsns.2012.06.001
- [18] B. Ahmad and S.K. Ntouyas, *An existence theorem for fractional hybrid differential inclusions of Hadamard type with Dirichlet boundary conditions*, Abstr. Appl. Anal. (2014), Art. ID 705809, 7 pages.
- [19] B.C. Dhage and S.K. Ntouyas, *Existence results for boundary value problems for fractional hybrid differential inclusions*, Topol. Methods Nonlinear Anal. **44** (2014), 229–238.
- [20] B. Ahmad, S.K. Ntouyas and A. Alsaedi, *Existence results for a system of coupled hybrid fractional differential equations*, The Scientific World Journal, Volume 2014, Article ID 426438, 6 pages.
- [21] B. Ahmad and S.K. Ntouyas, *An existence theorem for fractional hybrid differential inclusions of Hadamard type with Dirichlet boundary conditions*, Abstr. Appl. Anal. **2014** (2014), Article ID 705809, 7 pages.
- [22] B.C. Dhage, *A fixed point theorem in Banach algebras with applications to functional integral equations*, Kyungpook Math. J. **44** (2004), 145–155.
- [23] S. Sitho, S.K. Ntouyas and J. Tariboon, *Existence results for hybrid fractional integro-differential equations*, Bound. Value Prob. **2015** (2015) 113. doi:10.1186/s13661-015-0376-7
- [24] A. Lasota and Z. Opial, *An application of the Kakutani-Ky Fan theorem in the theory of ordinary differential equations*, Bull. Acad. Polon. Sci. Ser. Sci. Math. Astronom. Phys. **13** (1965), 781–786.

Received 24 September 2015

