

**OPTIMAL CONTROL OF GENERAL MCKEAN-VLASOV
STOCHASTIC EVOLUTION EQUATIONS ON HILBERT
SPACES AND NECESSARY CONDITIONS OF OPTIMALITY**

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Abstract

In this paper we consider controlled McKean-Vlasov stochastic evolution equations on Hilbert spaces. We prove existence and uniqueness of solutions and regularity properties thereof. We use relaxed controls, adapted to a current of sub-sigma algebras generated by observable processes, and taking values from a Polish space. We introduce an appropriate topology based on weak star convergence. We prove continuous dependence of solutions on controls with respect to appropriate topologies. These results are then used to prove existence of optimal controls for Bolza problems. Then we develop the necessary conditions of optimality based on semi-martingale representation theory on Hilbert spaces. Next we show that the adjoint processes arising from the necessary conditions of optimality can be constructed from the solution of certain BSDE.

Keywords: McKean-Vlasov stochastic differential equation, Hilbert spaces, relaxed controls, existence of optimal controls.

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