

EXPONENTIAL STABILITY OF NONLINEAR NON-AUTONOMOUS MULTIVARIABLE SYSTEMS

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Abstract

We consider nonlinear non-autonomous multivariable systems governed by differential equations with differentiable linear parts. Explicit conditions for the exponential stability are established. These conditions are formulated in terms of the norms of the derivatives and eigenvalues of the variable matrices, and certain scalar functions characterizing the nonlinearity. Moreover, an estimate for the solutions is derived. It gives us a bound for the region of attraction of the steady state. As a particular case we obtain absolute stability conditions.

Our approach is based on a combined usage of the properties of the "frozen" Lyapunov equation, and recent norm estimates for matrix functions. An illustrative example is given.

Keywords: nonlinear nonautonomous systems, exponential stability, absolute stability.

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