

**CONTROLLABILITY FOR SOME PARTIAL FUNCTIONAL
INTEGRODIFFERENTIAL EQUATIONS WITH NONLOCAL
CONDITIONS IN BANACH SPACES**

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Abstract

This work concerns the study of the controllability of some partial functional integrodifferential equation with nonlocal initial conditions in Banach spaces. It gives sufficient conditions that ensure the controllability of the system by supposing that its linear homogeneous part admits a resolvent operator in the sense of Grimmer, and by making use of the measure of non-compactness and the Mönch fixed-point theorem. As a result, we obtain a generalization of the work of Y.K. Chang, J.J. Nieto and W.S. Li (*J. Optim. Theory Appl.* 142, 267–273 (2009)), without assuming the compactness of the resolvent operator. An example of application is given for illustration.

Keywords: controllability, integrodifferential equations, nonlocal initial condition, resolvent operator, Mönch fixed-point theorem.

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REFERENCES

- [1] A.A. Agrachev, (ed.), *Mathematical Control Theory. Summer School on Mathematical Control Theory*, First Edition, The Abdus Salam ICTP Publications and Printing Section (2002).
- [2] R. Atmania and S. Mazouzi, *Controllability of semilinear integrodifferential equations with nonlocal conditions*, *Electr. J. Diff. Equ.* **2005** (75) (2005) 1–9.
- [3] K. Balachandran and J.Y. Park, *Existence of solutions and controllability of nonlinear integrodifferential systems in Banach spaces*, *Mathematical Problems in Engineering* **2** (2003) 65–79. doi:10.1155/S1024123X03201022.
- [4] J. Banaś and K. Goebel, *Measures of Noncompactness in Banach Spaces*, *Lecture Notes in Pure and Applied Mathematics*, vol. 60 (Marcel Dekker, New York, 1980).
- [5] L. Byszewski, *Theorems about the existence and uniqueness of solutions of a semilinear evolution nonlocal Cauchy problem*, *J. Math. Anal. Appl.* **162** (1991) 494–505. doi:10.1016/0022-247X(91)90164-U.
- [6] B. Cahlon, D.M. Kulkarni and P. Shi, *Stepwise stability for the heat equation with a nonlocal constraint*, *Siam J. Numer. Anal.* **32** (2) (1995) 571–593. doi:10.1137/0732025.
- [7] Y.K. Chang, J.J. Nieto and W.S. Li, *Controllability of semilinear differential systems with nonlocal initial conditions in Banach spaces*, *J. Optim. Theory Appl.* **142** (2009) 267–273. doi:10.1007/s10957-009-9535-2.
- [8] M.C. Delfour and J.P. Zolésio, *Shapes and Geometries: Metrics, Analysis, Differential Calculus, and Optimization*, *SIAM series on Advances in Design and Control*, Society for Industrial and Applied Mathematics (Philadelphia, Second Edition, 2011). doi:10.1137/1.9780898719826.
- [9] W. Desch, R. Grimmer and W. Schappacher, *Some considerations for linear integrodifferential equations*, *J. Math. Anal. and Appl.* **104** (1984) 219–234. doi:10.1016/0022-247X(84)90044-1.
- [10] W. Desch, R. Grimmer and W. Schappacher, *Well-posedness and wave propagation for a class of integrodifferential equations in Banach space*, *J. Differ. Equ.* **74** (2) (1988) 391–411. doi:10.1016/0022-0396(88)90011-3.
- [11] K. Ezzinbi, H. Toure and I. Zabsonre, *Existence and regularity of solutions for some partial functional integrodifferential equations in Banach spaces*, *Nonlin. Anal. TMA* **70** (2009) 2761–2771. doi:10.1016/j.na.2008.04.001.
- [12] R. Grimmer, *Resolvent operators for integral equations in a Banach space*, *AMS* **273** (1982), 333–349. doi:10.1090/S0002-9947-1982-0664046-4.
- [13] J. Liang, J.H. Liu and Xiao Ti-Jun, *Nonlocal problems for integrodifferential equations*, *DCDIS Series A: Math. Anal.* **15** (2008) 815–824.
- [14] H. Mönch, *Boundary value problems for nonlinear ordinary differential equations of second order in Banach spaces*, *Nonlin. Anal. TMA* **4** (5) (1980) 985–999.

- [15] P. Neittaanmäki and D. Tiba, *Optimal Control of Nonlinear Parabolic Systems: Theory, Algorithms and Applications*, Pure and Applied Mathematics, A series of Monographs and Textbooks, 179 (1994).
- [16] M.D. Quinn and N. Carmichael, *An approach to nonlinear control problem using fixed point methods, degree theory and pseudo-inverses*, Numer. Funct. Anal. Optim. **7** (1984) 197–219. doi:10.1080/01630568508816189.
- [17] S. Selvi and M.M. Arjunan, *Controllability results for impulsive differential systems with finite delay*, J. Nonlin. Sci. Appl. **5** (2012) 206–219.
- [18] M. Schulz, *Control Theory in Physics and Other Fields of Science: Concepts, Tools and Applications*, Springer Tracts in Modern Physics, 215 (2006).
- [19] I.I. Vrabie, *C_0 -Semigroups and Applications*, Mathematics Studies. **191** (2003).
- [20] J. Wang, Z. Fan and Y. Zhou, *Nonlocal controllability of semilinear dynamic systems with fractional derivative in Banach spaces*, J. Optim. Theory Appl. **154** (2012) 292–302. doi:10.1007/s10957-012-9999-3.
- [21] J. Wang and W. Wei, *Controllability of integrodifferential systems with nonlocal initial conditions in Banach spaces*, J. Math. Sci. **177** (3) (2011) 459–465. doi:10.1007/s10958-011-0471-y.

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