

$(k - 1)$ -KERNELS IN STRONG k -TRANSITIVE DIGRAPHS

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Abstract

Let $D = (V(D), A(D))$ be a digraph and $k \geq 2$ be an integer. A subset N of $V(D)$ is k -independent if for every pair of vertices $u, v \in N$, we have $d(u, v) \geq k$; it is l -absorbent if for every $u \in V(D) - N$, there exists $v \in N$ such that $d(u, v) \leq l$. A (k, l) -kernel of D is a k -independent and l -absorbent subset of $V(D)$. A k -kernel is a $(k, k - 1)$ -kernel.

A digraph D is k -transitive if for any path $x_0x_1 \cdots x_k$ of length k , x_0 dominates x_k . Hernández-Cruz [3-*transitive digraphs*, *Discuss. Math. Graph Theory* **32** (2012) 205–219] proved that a 3-transitive digraph has a 2-kernel if and only if it has no terminal strong component isomorphic to a 3-cycle. In this paper, we generalize the result to strong k -transitive digraphs and prove that a strong k -transitive digraph with $k \geq 4$ has a $(k - 1)$ -kernel if and only if it is not isomorphic to a k -cycle.

Keywords: digraph, transitive digraph, k -transitive digraph, k -kernel.

2010 Mathematics Subject Classification: 05C20.

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Received 7 January 2014

Revised 15 May 2014

Accepted 19 May 2014