

ON \bullet -LINE SIGNED GRAPHS $L_\bullet(S)$

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Abstract

A *signed graph* (or *sigraph* for short) is an ordered pair $S = (S^u, \sigma)$, where S^u is a graph, $G = (V, E)$, called the underlying graph of S and $\sigma : E \rightarrow \{+, -\}$ is a function from the edge set E of S^u into the set $\{+, -\}$. For a sigraph S its \bullet -line sigraph, $L_\bullet(S)$ is the sigraph in which the edges of S are represented as vertices, two of these vertices are defined adjacent whenever the corresponding edges in S have a vertex in common, any such L -edge ee' has the sign given by the product of the signs of the edges incident with the vertex in $e \cap e'$. In this paper we establish a structural characterization of \bullet -line sigraphs, extending a well known characterization of line graphs due to Harary. Further we study several standard properties of \bullet -line sigraphs, such as the balanced \bullet -line sigraphs, sign-compatible \bullet -line sigraphs and \mathcal{C} -sign-compatible \bullet -line sigraphs.

Keywords: sigraph, line graph, \bullet -line sigraph, balance, sign-compatibility, \mathcal{C} -sign-compatibility.

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