

## ON THE PROPERTIES OF THE GENERALIZED NORMAL DISTRIBUTION

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### Abstract

The target of this paper is to provide a critical review and to enlarge the theory related to the Generalized Normal Distributions (GND). This three term (position, scale shape) distribution is based in a strong theoretical background due to Logarithm Sobolev Inequalities. Moreover, the GND is the appropriate one to support the Generalized entropy type Fisher's information measure.

**Keywords:** entropy type Fisher's information, Shannon entropy, Normal distribution, truncated distribution.

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### REFERENCES

- [1] W.A. Benter, *Generalized Poincaré inequality for the Gaussian measures*, Amer. Math. Soc. **105** (2) (1989) 49–60.
- [2] T. Cacoullos and M. Koutras, *Quadric forms in spherical random variables: Generalized non-central  $\chi^2$  distribution*, Naval Research Logistics Quarterly **31** (1984) 447–461. doi:10.1002/nav.3800310310
- [3] T.M. Cover and J.A. Thomas, *Elements of Information Theory*, 2nd Ed. (Wiley, 2006).
- [4] K. Fragiadakis and S.G. Meintanis, *Test of fit for asymmetric Laplace distributions with applications*, J. of Statistics: Advances in Theory and Applications **1** (1) (2009) 49–63.

- [5] E. Gómez, M.A. Gómez-Villegas and J.M. Marin, *A multivariate generalization of the power exponential family of distributions*, *Comm. Statist. Theory Methods* **27** (3) (1998) 589–600. doi:10.1080/03610929808832115
- [6] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series, and Products* (Elsevier, 2007).
- [7] L. Gross, *Logarithm Sobolev inequalities*, *Amer. J. Math.* **97** (761) (1975) 1061–1083. doi:10.2307/2373688
- [8] C.P. Kitsos and N.K. Tavoularis, *Logarithmic Sobolev inequalities for information measures*, *IEEE Trans. Inform. Theory* **55** (6) (2009) 2554–2561. doi:10.1109/TIT.2009.2018179
- [9] C.P. Kitsos and N.K. Tavoularis, *New entropy type information measures*, in: *Information Technology Interfaces (ITI 2009)*, Luzar, Jarec and Bekic (Ed(s)), (Dubrovnic, Croatia, 2009) 255–259.
- [10] C.P. Kitsos and T.L. Toulías, *Bounds for the generalized entropy-type information measure*, *J. Comm. Comp.* **9** (1) (2012) 56–64.
- [11] C.P. Kitsos, T.L. Toulías and C.P. Trandafir, *On the multivariate  $\gamma$ -ordered normal distribution*, *Far East J. of Theoretical Statistics* **38** (1) (2012) 49–73.
- [12] C.P. Kitsos and T.L. Toulías, *On the family of the  $\gamma$ -ordered normal distributions*, *Far East J. of Theoretical Statistics* **35** (2) (2011) 95–114.
- [13] C.P. Kitsos and T.L. Toulías, *New information measures for the generalized normal distribution*, *Information* **1** (2010) 13–27. doi:10.3390/info1010013
- [14] C.P. Kitsos and T.L. Toulías, *Entropy inequalities for the generalized Gaussia*, in: *Information Technology Interfaces (ITI 2010)*, (Ed(s)), (Cavtat, Croatia, 2010) 551–556.
- [15] S. Kotz, *Multivariate distribution at a cross-road*, in: *Statistical Distributions in Scientific Work Vol. 1*, Patil, Kotz, Ord (Ed(s)), (Dordrecht, The Netherlands: D. Reidel Publ., 1975) 247–270. doi:10.1007/978-94-010-1842-5\_20
- [16] S. Kullback and A. Leibler A, *On information and sufficiency*, *Ann. Math. Statist.* **22** (1951) 79–86. doi:10.1214/aoms/1177729694
- [17] S. Nadarajah, *A generalized normal distribution*, *J. Appl. Stat.* **32** (7) (2005) 685–694. doi:10.1080/02664760500079464
- [18] S. Nadarajah, *The Kotz type distribution with applications*, *Statistics* **37** (4) (2003) 341–358. doi:10.1080/0233188031000078060
- [19] T. Papaioanou and K. Ferentinos, *Entropic descriptor of a complex behaviour*, *Comm. Stat. Theory and Methods* **34** (2005) 1461–1470.
- [20] M. Del Pino, J. Dolbeault and I. Gentil, *Nonlinear difussions, hypercontractivity and the optimal  $L^p$ -Euclidean logarithmic Sobolev inequality*, *J. Math. Anal. Appl.* **293** (2) (2004) 375–388. doi:10.1016/j.jmaa.2003.10.009

- [21] C.E. Shannon, *A mathematical theory of communication*, Bell Syst. Tech. J. **27** (1948) 379–423, 623–656. doi:j.1538-7305.1948.tb01338.x
- [22] S. Sobolev, *On a theorem of functional analysis*, English translation: AMS Transl. Ser. 2 **34** (1963) 39–68.

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