

EXTREMAL (IN)DEPENDENCE OF A MAXIMUM AUTOREGRESSIVE PROCESS

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Abstract

Maximum autoregressive processes like MARMA (Davis and Resnick, [5] 1989) or power MARMA (Ferreira and Canto e Castro, [12] 2008) have singular joint distributions, an unrealistic feature in most applications. To overcome this pitfall, absolute continuous versions were presented in Alpuim and Athayde [2] (1990) and Ferreira and Canto e Castro [14] (2010b), respectively. We consider an extended version of absolute continuous maximum autoregressive processes that accommodates both asymptotic tail dependence and independence. A full characterization of the bivariate lag- m tail dependence is presented. This will be useful in an adjustment procedure of the model to real data. An illustration with financial data is presented at the end.

Keywords: extreme value theory, autoregressive processes, tail dependence, asymptotic tail independence.

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