

ON THE TAIL INDEX ESTIMATION OF AN AUTOREGRESSIVE PARETO PROCESS

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Abstract

In this paper we consider an autoregressive Pareto process which can be used as an alternative to heavy tailed MARMA. We focus on the tail behavior and prove that the tail empirical quantile function can be approximated by a Gaussian process. This result allows to derive a class of consistent and asymptotically normal estimators for the shape parameter. We will see through simulation that the usual estimation procedure based on an i.i.d. setting may fall short of the desired precision.

Keywords: extreme value theory, autoregressive processes, tail index estimation.

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