

**EXISTENCE RESULTS FOR IMPULSIVE SEMILINEAR  
FRACTIONAL DIFFERENTIAL INCLUSIONS  
WITH DELAY IN BANACH SPACES**

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**Abstract**

In this paper, we introduce a new concept of mild solution of some class of semilinear fractional differential inclusions of order  $0 < \alpha < 1$ . Also we establish an existence result when the multivalued function has convex values. The result is obtained upon the nonlinear alternative of Leray-Schauder type.

**Keywords:** fractional calculus, caputo fractional derivative, multivalued map, differential inclusions, mild solution, fixed point.

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## REFERENCES

- [1] S. Abbas, M. Benchohra and G.M. N'Guérékata, *Topics in Fractional Differential Equations*, Springer, New York, 2012. doi:10.1007/978-1-4614-4036-9
- [2] R.P. Agarwal, M. Benchohra and S. Hamani, *Boundary value problems for fractional differential equations*, Adv. Stud. Contemp. Math. **16** (2008), 181–196.
- [3] A. Anguraj, P. Karthikeyan and G.M. N'Guérékata, *Nonlocal Cauchy problem for some fractional abstract differential equations in Banach spaces*, Commun. Math. Anal. **6** (2009).
- [4] D. Baleanu, K. Diethelm, E. Scalas and J.J. Trujillo, *Fractional Calculus Models and Numerical Methods*, World Scientific Publishing, New York, 2012.
- [5] M. Belmekki and M. Benchohra, *Existence results for fractional order semilinear functional differential equations*, Proc. A. Razmadze Math. Inst. **146** (2008), 9–20.
- [6] M. Belmekki, M. Benchohra and L. Górniewicz, *Semilinear functional differential equations with fractional order and infinite delay*, Fixed Point Th. **9** (2008), 423–439.
- [7] M. Benchohra, J.R. Graef and S. Hamani, *Existence results for boundary value problems with nonlinear fractional differential equations*, Appl. Anal. **87** (2008), 851–863. doi:10.1080/00036810802307579
- [8] M. Benchohra, J. Henderson, S.K. Ntouyas and A. Ouahab, *Existence results for fractional order functional differential equations with infinite delay*, J. Math. Anal. Appl. **338** (2008), 1340–1350. doi:10.1016/j.jmaa.2007.06.021
- [9] M. Benchohra, S. Litimein and G. N'Guérékata, *On fractional integro-differential inclusions with state-dependent delay in Banach spaces*, Appl. Anal. **92** (2013), 335–350. doi:10.1080/00036811.2011.616496
- [10] C. Chen and M. Li, *On fractional resolvent operator functions* Semigroup Forum. **80** (2010), 121–142. doi:10.1007/s00233-009-9184-7
- [11] C-Cuevas and J-C. de Souza, *S-asymptotically W-periodic solutions of semilinear fractional integro-differential equations*, Appl. Math. Lett. **22** (2009), 865–870. doi:10.1016/j.aml.2008.07.013
- [12] C-Cuevas and J-C. de Souza, *Existence of S-asymptotically W-periodic solutions of fractional order functional integro-differential equations with infinite delay*, Nonlinear Anal. **72** (2010), 1680–1689. doi:10.1016/j.na.2009.09.007
- [13] A. Granas and J. Dugundji, *Fixed Point Theory*, Springer-Verlag, New York, 2003. doi:10.1007/978-0-387-21593-8
- [14] R. Hilfer, *Applications of Fractional Calculus in Physics*, World Scientific, Singapore, 2000.
- [15] A.A. Kilbas, Hari M. Srivastava and Juan J. Trujillo, *Theory and Applications of Fractional Differential Equations*, North-Holland Mathematics Studies, 204. Elsevier Science B.V., Amsterdam, 2006.

- [16] V. Lakshmikantham, S. Leela and J. Vasundhara, *Theory of Fractional Dynamic Systems*, Cambridge Academic Publishers, Cambridge, 2009.
- [17] A. Lasota and Z. Opial, *An application of the Kakutani-Ky Fan theorem in the theory of ordinary differential equations*, Bull. Acad. Pol. Sci. Ser. Sci. Math. Astronom. Phys. **13** (1965), 781–786.
- [18] M. Li and Q. Zheng, *On spectral inclusions and approximations of  $\alpha$ -times resolvent families*, Semigroup Forum. **69** (2004), 356–368.
- [19] K.S. Miller and B. Ross, *An Introduction to the Fractional Calculus and Differential Equations*, John Wiley, New York, 1993.
- [20] I. Podlubny, *Fractional Differential Equations*, Academic Press, San Diego, 1999.
- [21] S.G. Samko, A.A. Kilbas and O.I. Marichev, *Fractional Integrals and Derivatives. Theory and Applications*, Gordon and Breach, Yverdon, 1993.
- [22] V.E. Tarasov, *Fractional Dynamics. Applications of Fractional Calculus to Dynamics of Particles, Fields and Media*, Springer, Heidelberg, 2010.

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