

## DIFFERENTIAL INCLUSIONS AND MULTIVALUED INTEGRALS

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### Abstract

In this paper we consider the nonlocal (nonstandard) Cauchy problem for differential inclusions in Banach spaces

$$x'(t) \in F(t, x(t)), \quad x(0) = g(x), \quad t \in [0, T] = I.$$

Investigation over some multivalued integrals allow us to prove the existence of solutions for considered problem. We concentrate on the problems for which the assumptions are expressed in terms of the weak topology in a Banach space. We recall and improve earlier papers of this type. The paper is complemented by a short survey about multivalued integration including Pettis and Henstock-Kurzweil-Pettis multivalued integrals.

**Keywords:** nonlocal Cauchy problem, Aumann integrals, Pettis integrals, Henstock-Kurzweil-Pettis integrals, measure of weak noncompactness.

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