

**THE EXOCENTER AND TYPE DECOMPOSITION
OF A GENERALIZED PSEUDOEFFECT ALGEBRA**

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Abstract

We extend the notion of the exocenter of a generalized effect algebra (GEA) to a generalized pseudoeffect algebra (GPEA) and show that elements of the exocenter are in one-to-one correspondence with direct decompositions of the GPEA; thus the exocenter is a generalization of the center of a pseudoeffect algebra (PEA). The exocenter forms a boolean algebra and the central elements of the GPEA correspond to elements of a sublattice of the exocenter which forms a generalized boolean algebra. We extend the notion of central orthocompleteness to GPEA, prove that the exocenter of a centrally orthocomplete GPEA (COGPEA) is a complete boolean algebra and show that the sublattice corresponding to the center is a complete boolean subalgebra. We also show that in a COGPEA, every element admits an exocentral cover and that the family of all exocentral covers, the so-called exocentral cover system, has the properties of a hull system on a generalized effect algebra. We extend the notion of type determining (TD) sets, originally introduced for effect algebras and then extended to GEAs

and PEAs, to GPEAs, and prove a type-decomposition theorem, analogous to the type decomposition of von Neumann algebras.

Keywords: pseudoeffect algebra, generalized pseudoeffect algebra, center, exocenter, central orthocompleteness, type determining set, type decomposition.

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REFERENCES

- [1] P. Bush, P. Lahti and P. Mittelstaedt, *The Quantum Theory of Measurement* (Lecture Notes in Physics, Springer, Berlin-Heidelberg-New York, 1991).
- [2] J.C Carrega, G. Chevalier and R. Mayet, *Direct decomposition of orthomodular lattices*, Alg. Univers. **27** (1990) 480–496. doi:10.1007/DF01188994
- [3] A. Dvurečenskij, *Central elements and Cantor-Bernstein’s theorem for pseudo-effect algebras*, J. Aust. Math. Soc. **74** (2003) 121–143.
- [4] A. Dvurečenskij and S. Pulmannová, *New Trends in Quantum Structures* (Kluwer, Dordrecht, 2000).
- [5] A. Dvurečenskij and T. Vetterlein, *Pseudoeffect algebras I. Basic properties*, Int. J. Theor. Phys. **40** (2001) 685–701. doi:10.1023/A:1004192715509
- [6] A. Dvurečenskij and T. Vetterlein, *Pseudoeffect algebras II. Group representations*, Int. J. Theor. Phys. **40** (2001) 703–726. doi:10.1023/A:1004144832348
- [7] A. Dvurečenskij and T. Vetterlein, *Algebras in the positive cone of po-groups*, Order **19** (2002) 127–146. doi:10.1023A:1016551707-476
- [8] A. Dvurečenskij and T. Vetterlein, *Generalized pseudo-effect algebras*, in: *Lectures on Soft Computing and Fuzzy Logic; Adv. Soft Comput.*, (Ed(s)), (Physica, Heidelberg, 2001) 89–111.
- [9] D.J. Foulis and M.K. Bennett, *Effect algebras and unsharp quantum logics*, Found. Phys. **24** (1994) 1331–1352. doi:10.1007/BF02283036
- [10] D.J. Foulis and S. Pulmannová, *Type-decomposition of an effect algebra*, Found. Phys. **40** (2010) 1543–1565. doi:10.1007/s10701-009-9344-3
- [11] D.J. Foulis and S. Pulmannová, *Centrally orthocomplete effect algebras*, Algebra Univ. **64** (2010) 283–307. doi:10.1007/s00012-010-0100-5
- [12] D.J. Foulis and S. Pulmannová, *Hull mappings and dimension effect algebras*, Math. Slovaca **61** (2011) 485–522. doi:10.2478/s12175-011-0025-2
- [13] D.J. Foulis and S. Pulmannová, *The exocenter of a generalized effect algebra*, Rep. Math. Phys. **61** (2011) 347–371.
- [14] D.J. Foulis and S. Pulmannová, *The center of a generalized effect algebra*, to appear in *Demonstratio Math.*

- [15] D.J. Foulis and S. Pulmannová, *Hull determination and type decomposition for a generalized effect algebra*, Algebra Univers. **69** (2013) 45–81.
doi:10.1007/s00012-012-0214-z
- [16] D.J. Foulis, S. Pulmannová and E. Vinceková, *Type decomposition of a pseudoeffect algebra*, J. Aust. Math. Soc. **89** (2010) 335–358.
doi:10.1017/S1446788711001042
- [17] K.R. Goodearl and F. Wehrung, *The Complete Dimension Theory of Partially Ordered Systems with Equivalence and Orthogonality*, Mem. Amer. Math. Soc. **831** (2005).
- [18] R.J. Greechie, D.J. Foulis and S. Pulmannová, *The center of an effect algebra*, Order **12** (1995) 91–106. doi:10.1007/BF01108592
- [19] G. Grätzer, *General Lattice Theory* (Academic Press, New York, 1978).
- [20] J. Hedlíková and S. Pulmannová, *Generalized difference posets and orthoalgebras*, Acta Math. Univ. Comenianae **45** (1996) 247–279.
- [21] M.F. Janowitz, *A note on generalized orthomodular lattices*, J. Natur. Sci and Math. **8** (1968) 89–94.
- [22] G. Jenča, *Subcentral ideals in generalized effect algebras*, Int. J. Theor. Phys. **39** (2000) 745–755. doi:10.1023/A:1003610426013
- [23] G. Kalmbach, *Measures and Hilbert Lattices* (World Scientific Publishing Co., Singapore, 1986).
- [24] G. Kalmbach and Z. Riečanová, *An axiomatization for abelian relative inverses*, Demonstratio Math. **27** (1994) 535–537.
- [25] F. Kôpka and F. Chovanec, *D-posets*, Math. Slovaca **44** (1994) 21–34.
- [26] L.H. Loomis, *The Lattice-Theoretic Background of the Dimension Theory of Operator Algebras* (Mem. Amer. Math. Soc. No. 18, 1955).
- [27] S. Maeda, *Dimension functions on certain general lattices*, J. Sci. Hiroshima Univ **A 19** (1955) 211–237.
- [28] A. Mayet-Ippolito, *Generalized orthomodular posets*, Demonstratio Math. **24** (1991) 263–274.
- [29] F.J. Murray and J. von Neumann, *On Rings of Operators*, J. von Neumann collected works, vol. III, Pergamon Press, Oxford, 1961, 6–321.
- [30] S. Pulmannová and E. Vinceková, *Riesz ideals in generalized effect algebras and in their unitizations*, Algebra Univ. **57** (2007) 393–417.
doi:10.1007/s00012-007-2043-z
- [31] S. Pulmannová and E. Vinceková, *Abelian extensions of partially ordered partial monoids*, Soft Comput. **16** (2012) 1339–1346.
doi:10.1007/s00500-012-0814-8
- [32] A. Ramsay, *Dimension theory in complete weakly modular orthocomplemented lattices*, Trans. Amer. Math. Soc. **116** (1965) 9–31.

- [33] Z. Riečanová, *Subalgebras, intervals, and central elements of generalized effect algebras*, Int. J. Theor. Phys. **38** (1999) 3209–3220.
doi:10.1023/A:1026682215765
- [34] M.H. Stone, *Postulates for Boolean algebras and generalized Boolean algebras*, Amer. J. Math. **57** (1935) 703–732.
- [35] A. Wilce, *Perspectivity and congruence in partial Abelian semigroups*, Math. Slovaca **48** (1998) 117–135.
- [36] Y. Xie and Y. Li, *Riesz ideals in generalized pseudo effect algebras and in their unitizations*, Soft Comput. **14** (2010) 387–398.
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