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# AN APPROACH TO DISTRIBUTION OF THE PRODUCT OF TWO NORMAL VARIABLES

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**Abstract**

The distribution of product of two normally distributed variables come from the first part of the XX Century. First works about this issue were [1] and [2] showed that under certain conditions the product could be considered as a normally distributed.

A more recent approach is [3] that studied approximation to density function of the product using three methods: numerical integration, Monte Carlo simulation and analytical approximation to the result using the normal distribution. They showed as the inverse variation coefficient $\frac{\mu}{\sigma}$ increases, the distribution of the product of two independent normal variables tends towards a normal distribution.

Our study is focused in Ware and Lad approaches. The objective was studying which factors have more influence in the presence of normality for the product of two independent normal variables. We have considered two factors: the inverse of the variation coefficient value $\frac{\mu}{\sigma}$ and the combined ratio (product of the two means divided by standard deviation): $\frac{\mu_1\mu_2}{\sigma}$ for two normal variables with the same variance.

Our results showed that for low values of the inverse of the variation coefficient (less than 1) normal distribution is not a good approximation for the product. Another one, influence of the combined ratio value is less than influence of the inverse of coefficients of variation value.

**Keywords:** product of normally distributed variables, inverse coefficient of variation, numerical integration, Monte Carlo simulation, combined ratio.

**2010 Mathematics Subject Classification:** 62E17, 62E10.

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