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VON BERTALANFFY'S GROWTH DYNAMICS WITH STRONG ALLEE EFFECT

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Abstract

Von Bertalanffy's model is one of the most popular differential equation used in order to study the increase in average length or weight of fish. However, this model does not include demographic Allee effect. This phenomenon is known in the fisheries literature as "depensation", which arises when populations decline rapidly at low densities. In this paper we develop and investigate new corrected von Bertalanffy's models with Allee effects. The generalization that we propose results from considering correction factors, one of rational type and the others of polynomial type, where two parameters are considered. The use of a parameter $C > 0$ leads the presented generalization, which yields some more flexible models with variable extinction rates. An Allee limit or unstable equilibrium E^u is incorporated so that the models under study have strong Allee effect. We analyze and show the transition from the strong Allee effect to the inexistence of this effect, through by a "weakening" of the Allee effect, depending on the parameters C and E^u. Finally, we discuss the flexibility of corrected von Bertalanffy's sigmoid growth curves. So, the correspondents inflexion points are variable, i.e., the fish mass when the growth rate is maximum may be varied.

Keywords: Von Bertalanffy's model, population dynamics, Allee effect.

2010 Mathematics Subject Classification: 92D25, 92D99, 92D40.

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