

## ON THE SOLIDITY OF GENERAL VARIETIES OF TREE LANGUAGES

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### Abstract

For a class of hypersubstitutions  $\mathcal{K}$ , we define the  $\mathcal{K}$ -solidity of general varieties of tree languages (GVTLs) that contain tree languages over all alphabets, general varieties of finite algebras (GVFAs), and general varieties of finite congruences (GVFCs). We show that if  $\mathcal{K}$  is a so-called category of substitutions, a GVTL is  $\mathcal{K}$ -solid exactly in case the corresponding GVFA, or the corresponding GVFC, is  $\mathcal{K}$ -solid. We establish the solidity status of several known GVTLs with respect to certain categories of substitutions derived from some important classes of tree homomorphisms.

**Keywords:** varieties of tree languages, solid varieties, hypersubstitutions, tree homomorphisms.

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