

STOCHASTIC DIFFERENTIAL EQUATIONS ON BANACH SPACES AND THEIR OPTIMAL FEEDBACK CONTROL

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Abstract

In this paper we consider stochastic differential equations on Banach spaces (not Hilbert). The system is semilinear and the principal operator generating a C_0 -semigroup is perturbed by a class of bounded linear operators considered as feedback operators from an admissible set. We consider the corresponding family of measure valued functions and present sufficient conditions for weak compactness. Then we consider applications of this result to several interesting optimal feedback control problems. We present results on existence of optimal feedback operators.

Keywords: stochastic differential equations, Banach spaces, optimal feedback control, objective functionals, Lévy-Prohorov metric, Hausdorff dimension, time-optimal problems.

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