

**NONLINEAR FRACTIONAL DIFFERENTIAL INCLUSIONS
WITH ANTI-PERIODIC TYPE INTEGRAL BOUNDARY
CONDITIONS**

BASHIR AHMAD

Department of Mathematics
Faculty of Science
King Abdulaziz University
P.O. Box 80203, Jeddah 21589, Saudi Arabia
e-mail: bashir_qau@yahoo.com

AND

SOTIRIS K. NTOUYAS

Department of Mathematics
University of Ioannina
451 10 Ioannina, Greece
e-mail: sntouyas@uoi.gr

Abstract

This article studies a boundary value problem of nonlinear fractional differential inclusions with anti-periodic type integral boundary conditions. Some existence results are obtained via fixed point theorems. The cases of convex-valued and nonconvex-valued right hand sides are considered. Several new results appear as a special case of the results of this paper.

Keywords: fractional differential inclusions, anti-periodic, integral boundary conditions, existence, fixed point theorems.

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