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AN ABSTRACT CAUCHY PROBLEM FOR HIGHER ORDER FUNCTIONAL DIFFERENTIAL INCLUSIONS WITH INFINITE DELAY

Tran Dinh Ke

Department of Mathematics
Hanoi National University of Education
136 Xuan Thuy, Cau Giay, Hanoi, Vietnam

e-mail: ketd@hn.vnn.vn

Valeri Obukhovskii*

Faculty of Mathematics
Voronezh State University
Universitetskays pl. 1, 394006 Voronezh, Russia

e-mail: valerio-ob2000@mail.ru

Ngai-Ching Wong

Department of Applied Mathematics
National Sun Yat-sen University
Kaohsiung 804, Taiwan

e-mail: wong@math.nsysu.edu.tw

and

Jen-Chih Yao

Center for General Education
Kaohsiung Medical University
Kaohsiung 807, Taiwan

e-mail: yaojc@kmu.edu.tw

*Corresponding author.
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Abstract

The existence results for an abstract Cauchy problem involving a higher order differential inclusion with infinite delay in a Banach space are obtained. We use the concept of the existence family to express the mild solutions and impose the suitable conditions on the nonlinearity via the measure of noncompactness in order to apply the theory of condensing multimaps for the demonstration of our results. An application to some classes of partial differential equations is given.

Keywords: Cauchy problem, functional differential inclusion, infinite delay, higher order, existence family, phase space, fixed point, multivalued map, measure of noncompactness, condensing map.

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References

[1] W. Arendt, *Vector-valued Laplace transforms and Cauchy problems*, Israel J. Math. **59** (3) (1987), 327–352. doi:10.1007/BF02774144

[2] W. Arendt, C.J.K. Batty, M. Hieber and F. Neubrander, Vector-valued Laplace Transforms and Cauchy Problems, Monographs in Mathematics, 96. Birkhauser Verlag, Basel, 2001.

[3] J.-P. Aubin and H. Frankowska, Set-Valued Analysis, Reprint of the 1990 edition, Modern Birkhauser Classics, Birkhauser Boston, Inc., Boston, MA, 2009.

[4] Yu. G. Borisovich, B.D. Gelman, A.D. Myshkis and V.V. Obukhovskii, Introduction to the Theory of Multivalued Maps and Differential Inclusions, 2nd edition, Librokom, Moscow, 2011 (in Russian).

[5] G. Da Prato and E. Sinestrari, *Differential operators with nondense domain*, Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) **14** (2) (1987), 285–344.

[6] K. Deimling, Multivalued Differential Equations, de Gruyter Series in Nonlinear Analysis and Applications, 1. Walter de Gruyter, Berlin, 1992.

[7] R. deLaubenfels, *Integrated semigroups, C-semigroups and the abstract Cauchy problem*, Semigroup Forum **41** (1) (1990), 83–95. doi:10.1007/BF02573380

[8] R. deLaubenfels, *Entire solutions of the abstract Cauchy problem*, Semigroup Forum **42** (1) (1991), 83–105. doi:10.1007/BF02573409

[9] R. deLaubenfels, *Existence and uniqueness families for the abstract Cauchy problem*, J. London Math. Soc. **44** (2) (1991), 310–338. doi:10.1112/jlms/s2-44.2.310

[10] R. deLaubenfels, Existence families, functional calculi and evolution equations, Lecture Notes in Mathematics, 1570. Springer-Verlag, Berlin, 1994.

[11] K.-J. Engel and R. Nagel, One-Parameter Semigroups for Linear Evolution Equations, Graduate Texts in Mathematics, 194. Springer-Verlag, New York, 2000.

[12] H.O. Fattorini, Second Order Linear Differential Equations in Banach Spaces, North-Holland Mathematics Studies, 108. Notas de Matematica [Mathematical Notes], 99. North-Holland Publishing Co., Amsterdam, 1985.

[13] E.P. Gatsori, L. Gorniewicz, S.K. Ntouyas and G.Y. Sficas, *Existence results for semilinear functional differential inclusions with infinite delay*, Fixed Point Theory **6** (1) (2005), 47–58.

[14] C. Gori, V. Obukhovskii, M. Ragni and P. Rubbioni, *Existence and continuous dependence results for semilinear functional differential inclusions with infinite delay*, Nonlinear Anal. **51** (5) (2002), Ser. A: Theory Methods, 765–782.

[15] L. Górniewicz, Topological Fixed Point Theory of Multivalued Mappings, 2nd edition, Topological Fixed Point Theory and Its Applications, 4. Springer, Dordrecht, 2006.

[16] J.K. Hale and J. Kato, *Phase space for retarded equations with infinite delay*, Funkcial. Ekvac. **21** (1) (1978), 11–41.

[17] M. Hieber, *Integrated semigroups and differential operators on L^p spaces*, Math. Ann. **291** (1) (1991), 1–16. doi:10.1007/BF01445187

[18] Y. Hino, S. Murakami and T. Naito, Functional Differential Equations with Infinite Delay, Lecture Notes in Mathematics, Vol. 1473, Springer-Verlag, Berlin-Heidelberg-New York, 1991.

[19] S. Hu and N.S. Papageorgiou, Handbook of multivalued analysis, Vol. I. Theory, Mathematics and its Applications, 419, Kluwer Academic Publishers, Dordrecht, 1997.

[20] C. Kaiser, *Integrated semigroups and linear partial differential equations with delay*, J. Math. Anal. Appl. **292** (2) (2004), 328–339. doi:10.1016/j.jmaa.2003.10.031

[21] M. Kamenskii, V. Obukhovskii and P. Zecca, Condensing Multivalued Maps and Semilinear Differential Inclusions in Banach Spaces, de Gruyter Series in Nonlinear Analysis and Applications, 7, Walter de Gruyter, Berlin-New York, 2001. doi:10.1515/9783110870893

[22] H. Kellerman and M. Hieber, *Integrated semigroups*, J. Funct. Anal. **84** (1) (1989), 160–180. doi:10.1016/0022-1236(89)90116-X

[23] M. Kisielewicz, Differential Inclusions and Optimal Control, Mathematics and its Applications (East European Series), 44. Kluwer Academic Publishers Group, Dordrecht; PWN—Polish Scientific Publishers, Warsaw, 1991.

[24] S.G. Krein, Linear Differential Equations in Banach Space, Translations of Mathematical Monographs, Vol. 29. American Mathematical Society, Providence, R.I., 1971.

[25] V. Lakshmikantham, L.Z. Wen and B.G. Zhang, Theory of Differential Equations With Unbounded Delay, Mathematics and its Applications, 298. Kluwer Academic Publishers Group, Dordrecht, 1994.

[26] J. Liang and T.J. Xiao, *Wellposedness results for certain classes of higher order abstract Cauchy problems connected with integrated semigroups*, Semigroup Forum **56** (1) (1998), 84–103. doi:10.1007/s00233-002-7007-1

[27] Y.C. Liou, V. Obukhovskii and J.C. Yao, *Controllability for a class of degenerate functional differential inclusions in a Banach space*, Taiwanese Journal of Math. **12** (8) (2008), 2179–2200.

[28] B. Liu, *Controllability of impulsive neutral functional differential inclusions with infinite delay*, Nonlinear Anal. **60** (8) (2005), 1533–1552. doi:10.1016/j.na.2004.11.022

[29] I.V. Mel'nikova and A.I. Filinkov, *Integrated semigroups and C-semigroups. Well-posedness and regularization of operator-differential problems*, (Russian) Uspekhi Mat. Nauk **49** (6) (1994), 111–150; English translation in Russian Math. Surveys **49** (6) (1994), 115–155.

[30] F. Neubrander, *Well-posedness of higher order abstract Cauchy problems*, Trans. Amer. Math. Soc. **295** (1) (1986), 257–290. doi:10.1090/S0002-9947-1986-0831199-8

[31] V. Obukhovskii and J.-C. Yao, *On impulsive functional differential inclusions with Hille-Yosida operators in Banach spaces*, Nonlinear Anal. **73** (6) (2010), 1715–1728. doi:10.1016/j.na.2010.05.009

[32] V. Obukhovskii and P. Zecca, *On semilinear differential inclusions in Banach spaces with nondensely defined operators*, J. Fixed Point Theory Appl. **9** (1) (2011), 85–100. doi:10.1007/s11784-011-0042-3

[33] A. Pazy, Semigroups of Linear Operators and Applications to Partial Differential Equations. Applied Mathematical Sciences, 44. Springer-Verlag, New York, 1983. doi:10.1007/978-1-4612-5561-1

[34] Y. Qin, Nonlinear Parabolic-Hyperbolic Coupled Systems and Their Attractors. Operator Theory: Advances and Applications, 184. Advances in Partial Differential Equations (Basel). Birkhauser Verlag, Basel, 2008.

[35] H.R. Thieme, *"Integrated semigroups" and integrated solutions to abstract Cauchy problems*, J. Math. Anal. Appl. **152** (2) (1990), 416–447. doi:10.1016/0022-247X(90)90074-P

[36] V.V. Vasil'ev, S.G. Krein and S.I. Piskarev, *Operator semigroups, cosine operator functions, and linear differential equations*, J. Soviet Math. **54** (4) (1991), 1042–1129. doi:10.1007/BF01138948

[37] T.-J. Xiao and J. Liang, The Cauchy Problem for Higher-Order Abstract Differential Equations, Lecture Notes in Mathematics, 1701. Springer-Verlag, Berlin, 1998.

[38] T. Xiao and J. Liang, *Differential operators and C-wellposedness of complete second order abstract Cauchy problems*, Pacific J. Math. **186** (1) (1998), 167–200. doi:10.2140/pjm.1998.186.167

[39] T.-J. Xiao and J. Liang, *Higher order abstract Cauchy problems: their existence and uniqueness families*, J. London Math. Soc. (2) **67** (1) (2003), 149–164.

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