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ON CONGRUENCE DISTRIBUTIVITY OF ORDERED ALGEBRAS WITH CONSTANTS

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Abstract

We define the order-congruence distributivity at 0 and order-congruence n-distributivity at 0 of ordered algebras with a nullary operation 0. These notions are generalizations of congruence distributivity and congruence n-distributivity. We prove that a class of ordered algebras with a nullary operation 0 closed under taking subalgebras and direct products is order-congruence distributive at 0 iff it is order-congruence n-distributive at 0. We also characterize such classes by a Mal'tsev condition.

Keywords: ordered algebra, n-distributivity, distributivity, Mal'tsev condition.

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