

## ALGEBRAIC STRUCTURE OF STEP NESTING DESIGNS

CÉLIA FERNANDES, PAULO RAMOS

*Área Científica de Matemática, Instituto Superior de Engenharia de Lisboa*  
*Rua Conselheiro Emídio Navarro, 01 1959–007 Lisboa, Portugal*

**e-mail:** cfernandes@deetc.isel.ipl.pt

**e-mail:** pramos@deetc.isel.ipl.pt

AND

JOÃO TIAGO MEXIA

*Departamento de Matemática*  
*Faculdade de Ciências e Tecnologia Universidade Nova de Lisboa*  
*Monte de Caparica 2829–516 Caparica, Portugal*

**e-mail:** jtm@fct.unl.pt

### Abstract

Step nesting designs may be very useful since they require fewer observations than the usual balanced nesting models. The number of treatments in balanced nesting design is the product of the number of levels in each factor. This number may be too large. As an alternative, in step nesting designs the number of treatments is the sum of the factor levels. Thus these models lead to a great economy and it is easy to carry out inference. To study the algebraic structure of step nesting designs we introduce the cartesian product of commutative Jordan algebras.

**Key Words:** commutative Jordan algebras, cartesian product of commutative Jordan algebras, step nesting, variance components, UMVUE.

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