

SEMI-ADDITIVE FUNCTIONALS AND COCYCLES IN THE CONTEXT OF SELF-SIMILARITY*

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Abstract

Kernel functions of stable, self-similar mixed moving averages are known to be related to nonsingular flows. We identify and examine here a new functional occurring in this relation and study its properties. To prove its existence, we develop a general result about semi-additive functionals related to cocycles. The functional we identify, is helpful when solving for the kernel function generated by a flow. Its presence also sheds light on the previous results on the subject.

Keywords and phrases: stable, self-similar processes with stationary increments, mixed moving averages, nonsingular flows, cocycles, semi-additive functionals.

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