

COMPARISON AT OPTIMAL LEVELS OF CLASSICAL  
TAIL INDEX ESTIMATORS: A CHALLENGE  
FOR REDUCED-BIAS ESTIMATION?\*

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*To Professor João Tiago Mexia, a token of friendship.*

**Abstract**

In this article, we begin with an asymptotic comparison at optimal levels of the so-called “*maximum likelihood*” (ML) extreme value index estimator, based on the excesses over a high random threshold, denoted PORT-ML, with PORT standing for *peaks over random thresholds*, with a similar ML estimator, denoted PORT-MP, with MP standing for *modified-Pareto*. The PORT-MP estimator is based on the same excesses, but with a trial of accommodation of bias on the Generalized Pareto model underlying those excesses. We next compare the behaviour of these ML implicit estimators with the equivalent behaviour of a few explicit tail index estimators, the *Hill*, the *moment*, the *generalized Hill* and the *mixed moment*. As expected, none of the estimators can always dominate the alternatives, even when we include second-order MVRB tail index estimators, with MVRB standing for *minimum-variance reduced-bias*. However, the asymptotic performance of the MVRB estimators is quite interesting and provides a challenge for a further study of these MVRB estimators at optimal levels.

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