

**EXTREMAL BEHAVIOUR OF STATIONARY
PROCESSES: THE CALIBRATION TECHNIQUE
IN THE EXTREMAL INDEX ESTIMATION**

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Abstract

Classical extreme value methods were derived when the underlying process is assumed to be a sequence of independent random variables. However when observations are taken along the time and/or the space the independence is an unrealistic assumption. A parameter that arises in this situation, characterizing the degree of local dependence in the extremes of a stationary series, is the extremal index, θ . In several areas such as hydrology, telecommunications, finance and environment, for example, the dependence between successive observations is observed so large values tend to occur in clusters. The extremal index is a quantity which, in an intuitive way, allows one to characterise the relationship between the dependence structure of the data and their extremal behaviour. Several estimators have been studied in the literature, but they endure a problem that usually appears in semiparametric estimators - a strong dependence on the high level u_n , with an increasing bias and a decreasing variance as the threshold decreases.

The calibration technique (Scheffé, 1973) is here considered as a procedure of controlling the bias of an estimator. It also leads to the construction of confidence intervals for the extremal index. A simulation study was performed for a stationary sequence and two sets of stationary data are under study for applying this technique.

Keywords: extreme value, stationary sequences, extremal index, estimation, calibration technique.

2000 Mathematics Subject Classification: 62G32, 62G09, 62G05, 62J05.

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Received 27 January 2010