

**AN ASYMPTOTICALLY UNBIASED  
MOMENT ESTIMATOR OF A NEGATIVE  
EXTREME VALUE INDEX\***

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**Abstract**

In this paper we consider a new class of consistent semi-parametric estimators of a negative extreme value index, based on the set of the  $k$  largest observations. This class of estimators depends on a control or tuning parameter, which enables us to have access to an estimator with a null second-order component of asymptotic bias, and with a rather interesting mean squared error, as a function of  $k$ .

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We study the consistency and asymptotic normality of the proposed estimators. Their finite sample behaviour is obtained through Monte Carlo simulation.

**Keywords:** extreme value index, semi-parametric estimation; moment estimator.

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